Rethinking Mutual Fund Performance: From Traditional Alpha to Achievable Alpha*

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Abstract

Mutual fund performance is traditionally evaluated using alpha, which measures the utility gain of an *unconstrained* investor who has access to the fund in addition to the benchmark factors. We prove that the utility gain of *shortsale-constrained* investors is instead measured by *achievable alpha*, estimated using only those factors with strictly positive weight in the shortsale-constrained benchmark-factor portfolio. Empirically, active-fund management is less valuable for constrained investors: while 62.54% of funds have positive traditional gross alpha, only 37.27% have positive achievable gross alpha for a benchmark containing eight Vanguard funds. Finally, achievable alphas significantly predict fund flows, particularly during market turmoil.

Keywords: Performance evaluation, Alpha, Value-added, Shortsale constraints, Market frictions.

JEL Classification: G11, G23.

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1 Introduction

The US mutual-fund industry manages more than 28 trillion dollars as of 2024. Of this amount, about 88 percent is held by retail investors (71 million US households, representing over 120 million individuals), with institutions, such as pension funds, insurance companies, and endowments, holding 12 percent (ICI Fact Book, 2025, p. 45). The average active mutual fund, however, underperforms its benchmark after fees,[1] and investment in active mutual funds has declined from 94% of assets under management in 1996 to only 41% in 2024 (American Century Investments, 2024, fig. 1). This highlights the importance of evaluating the performance of active mutual funds carefully.

Traditionally, mutual-fund performance is evaluated by the alpha obtained from regressing fund returns on benchmark-factor returns. The economic motivation for this metric stems from Gibbons, Ross, and Shanken (1989), who show that a quadratic form of the alpha measures the mean-variance utility improvement an investor can achieve when she has access to the fund, in addition to the benchmark factors. However, if the benchmark portfolio includes short positions in some factors, then for shortsale-constrained investors, this alpha is unachievable, and so is not an accurate measure of performance. In practice, many mutual-fund investors face shortsale impediments. For instance, most retail investors—who own 88 percent of mutual funds—do not take short positions due to share-borrowing costs, margin costs, or an aversion to the risk associated with short positions.[2] And, even institutional investors may face shortsale constraints. For example, the mandate of some pension funds precludes them from taking significant short positions directly or indirectly.[3] Even pension funds whose mandate allows them to short tend to hold only small short positions.[4]

[1] The Morningstar report by Armour, Jackson, Gorbatikov, and Kim (2024) shows that less than 22% of active strategies beat their passive counterparts over the ten years through 2024.

[2] For instance, Kelley and Tetlock (2017, p. 805) analyze a dataset with 144 billion dollars of retail trades and find that only 5.54% of the retail dollar volume corresponds to shortsales. Moreover, Gamble and Xu (2017) show that only around 1.2% of the retail trades in their dataset are short sales.

[3] For instance, the state of Georgia precludes its public pension funds from investing more than 5% of their assets in alternative investments such as hedge funds (Molk and Partnoy, 2019, p. 851).

[4] For example, (Molk and Partnoy, 2019, p. 853) explain that in a sample of private pension funds drawn from Fortune 1000 companies, “only 6–8% of assets were invested in hedge funds and other assets that could conceivably involve short selling activity.” Even hedge funds can face challenges when they short, including “legal and at times physical threats,” as described by Aliaj, Agnew, and Wiggins (2024).

In this paper, we propose a straightforward approach for evaluating mutual-fund performance for a shortsale-constrained investor. Theoretically, we demonstrate that the marginal utility improvement that a shortsale-constrained investor can achieve when she has access to a mutual fund can be measured by the *achievable alpha*, which is the fund alpha with respect to only those benchmark factors that have a strictly positive weight in the shortsale-constrained mean-variance portfolio. Intuitively, one would expect the achievable alpha to be larger than its traditional counterpart because excluding some factors from the benchmark portfolio should *worsen* its performance. However, we show theoretically that the achievable alpha can be *smaller* than the traditional alpha. To understand the intuition underlying this result, consider, for instance, the extreme example of a mutual fund that achieved a negative annual return of -2% over the evaluation period, but is evaluated with respect to a single benchmark factor that achieved a return of -3%. Then, the mutual-fund alpha is positive ($+1\%$) assuming a unit beta. But to benefit from this positive alpha, an investor must be able to short the benchmark factor. For an investor who cannot short the benchmark, the achievable alpha is only the fund's mean return, -2%.

For our empirical analysis, we consider six prominent benchmark factor models: the CAPM model of Sharpe (1964) and Lintner (1965), the four-factor model obtained by adding momentum to the three factors of Fama and French (1993) as in Carhart (1997), FFC, the five-factor model of Fama and French (2015), FF5, the six-factor model of Fama and French (2018), FF6, the four-factor model of Hou, Xue, and Zhang (2015), HXZ, and the five-factor model obtained by adding momentum to the four factors of Hou et al. (2015), HXZM. In addition, we consider a factor model containing the returns of eight US domestic equity Vanguard funds, VANG. Our motivation for considering this model arises from Berk and van Binsbergen (2015), who point out that the factors in prominent factor models do not account for transaction costs and, thus, do not represent the best alternative investment opportunity for mutual-fund investors. Consequently, they propose considering as a benchmark a set of 11 Vanguard funds that invest in domestic and foreign equities and are easily accessible to investors. Of these funds, we consider a factor model containing the returns of the eight funds that invest in only domestic equity.

We extend the argument of Berk and van Binsbergen (2015) and point out that five of the aforementioned models (FFC, FF5, FF6, HXZ, and HXZM) contain factors that are the returns of *long-short* portfolios, and thus, they are not easily accessible to shortsale-constrained investors. For instance, An, Huang, Lou, and Shi (2023, table 1) show that long-short mutual funds represent only a very small fraction (below 3%) of the assets under management in the mutual-fund industry. Moreover, they show that most mutual funds with short positions use a degree of leverage that is much smaller than that employed by the long-short factors in prominent asset-pricing models.[5] The limited shortselling of mutual funds could be explained by the recent findings of Johansson, Sabbatucci, and Tamoni (2025), who show that even without accounting for shorting fees, the short leg of factors such as the investment and profitability factors of Fama and French (2015) are difficult to implement in practice.[6] To address this concern, we consider the five aforementioned models using long-only versions of the factors in the original models.

Our main empirical finding is that mutual-fund performance measured in terms of the achievable alpha of a shortsale-constrained investor with respect to the long-only versions of the factor models is substantially *worse* than that measured in terms of traditional alpha with respect to long-short factor models. For instance, the top plot in Figure 1 shows that while the proportion of mutual funds with positive traditional gross-of-fees alpha with respect to the long-short factor models ranges from 51.21% for HXZ to 62.54% for VANG, the proportion of mutual funds with positive achievable gross-of-fees alpha with respect to the long-only models is only 11.81% for HXZ and 37.27% for VANG. These striking results are robust to measuring mutual-fund performance in terms of the value-added measure of Berk and van Binsbergen (2015).[7] For instance, the bottom plot in Figure 1 shows that while the proportion of mutual funds with positive traditional value-added with respect to the long-short factor models ranges from 35.12% for HXZ to 44.58% for VANG, the proportion

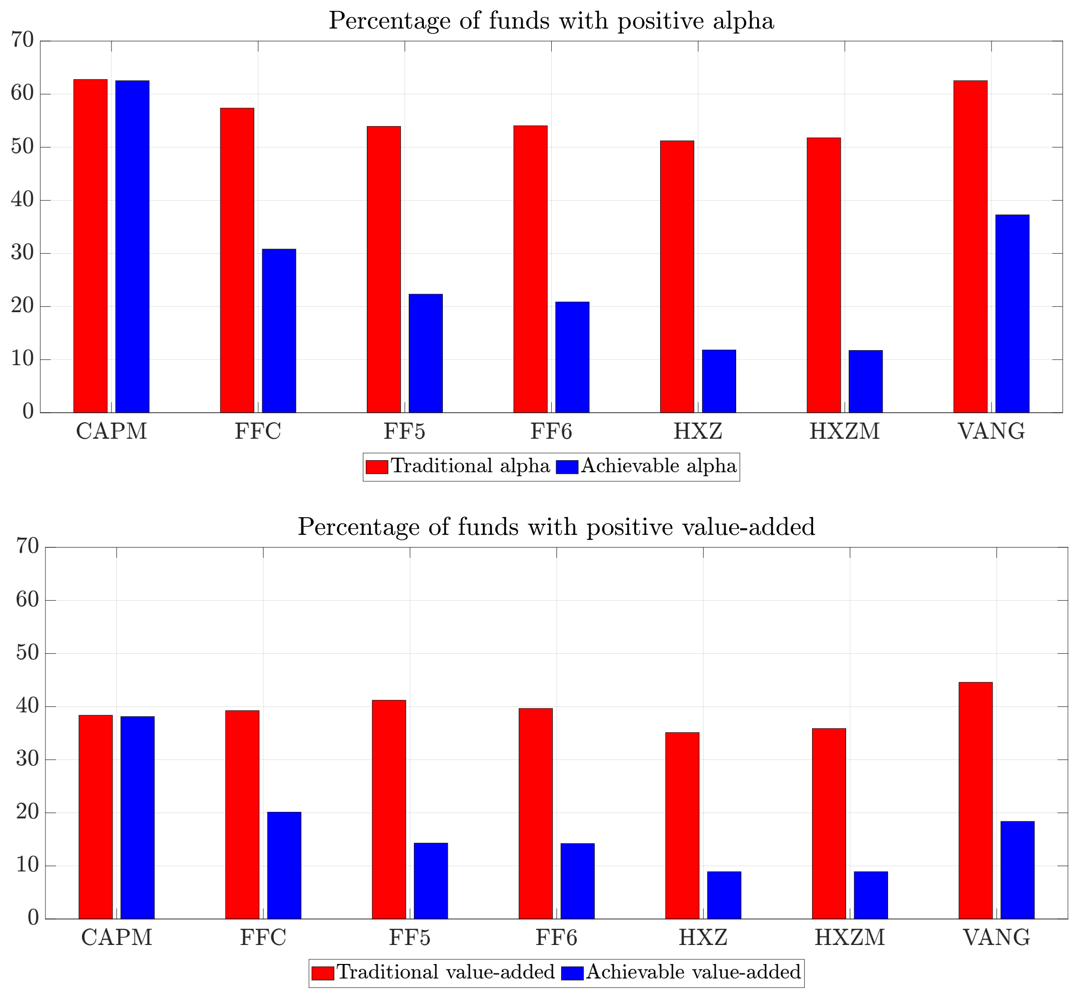
[5] The long-short factors in factor models typically have the same aggregate position in their long and short legs. In contrast, An et al. (2023, p. 2) show that only 8% of the mutual funds in their sample hold any short positions, and only 3% of the funds hold short positions that aggregate to more than 20% of their assets under management.

[6] Johansson et al. (2025) point out that the short leg of these factors includes stocks that "(i) display extreme exposure to the underlying factor characteristic and (ii) are not traded in practice, as revealed by fund holdings." They also mention there is a scarcity of funds investing in "losers," "weak profitability," and "aggressive" stocks required to replicate the academic factors.

[7] Berk and van Binsbergen (2015) explain the importance of measuring mutual-fund performance in terms of *value-added*, defined as the average of the product between a fund's gross abnormal returns and its assets under management. Following their approach, we define achievable value-added as the average of the product between achievable abnormal return and assets under management.

Figure 1: Traditional and achievable gross alpha and value-added

This figure depicts the proportion of funds with positive traditional and achievable gross alpha (top plot) and value-added (bottom plot) with respect to the seven factor models we consider. Traditional alphas are computed by regressing the fund returns on all long-short factors for each model, and achievable alphas on just those long-only factors with a strictly positive weight in the shortsale-constrained mean-variance portfolio (over the sample period for which we have return data for the fund). Traditional value-added is the average of the product of assets under management and abnormal returns, obtained by regressing fund returns on all long-short factors in each model, and achievable value-added is computed using fund abnormal returns with respect to just those long-only factors with a strictly positive weight in the shortsale-constrained mean-variance portfolio.



of mutual funds with positive achievable value-added with respect to the long-only models is 8.90% for HXZ and 18.38% for VANG.

The finding that mutual-fund performance *deteriorates* when evaluated using achievable alpha is counterintuitive. Both dropping some factors and using long-only versions of the remaining factors should worsen the performance of the benchmark portfolio, and thus, one would expect the alpha of a mutual fund to be larger with respect to the restricted benchmark. However, we show theoretically that if a fund has positive beta with respect to some of the factors with zero weight in the shortsale-constrained portfolio, then the achievable alpha can be smaller than the traditional alpha. The intuition is that benchmark factors with zero-weight in the shortsale-constrained portfolio *underperform* relative to the other factors. If a mutual fund has a positive beta with respect to such underperforming factors, then an investor would optimally wish to short the underperforming factors to hedge the risk of the mutual fund, which would allow her to take a larger position on the mutual fund. Empirically, we find that the long-only benchmark factors often have a zero weight in the mean-variance portfolio, and mutual funds often have a positive beta with respect to the zero-weight long-only factors. This implies that an unconstrained investor often wishes to short some of the benchmark factors against a positive position in the mutual fund. Thus, achievable alpha is often smaller than traditional alpha because the shortsale constraints limit the position that investors can take in the mutual fund.

To illustrate the intuition underlying our results, we consider in Section 3.3 a simple example of a benchmark with only two factors: the market (MKT) and the long-only version of momentum (UMD). The unconstrained mean-variance portfolio of these two factors assigns a weight of −0.80 to MKT and 1.40 to UMD, but the shortsale-constrained mean-variance portfolio assigns a weight of zero to MKT and 0.79 to UMD. As a result, we find that the achievable alpha of Columbia Acorn Fund (1.21%) is substantially smaller than the traditional alpha (1.79%). To understand why the achievable alpha is smaller than the traditional alpha, we compare the unconstrained and shortsale-constrained mean-variance portfolio weights of the two benchmark factors and the mutual fund. We find that an unconstrained investor would hold −1.15 in MKT, 0.91 in UMD, and 0.89 in the mutual fund. That is, the unconstrained investor would short the MKT to hedge the risk of the UMD factor and the mutual fund. In contrast, a shortsale-constrained investor would hold zero in MKT, 0.33 in UMD, and only 0.53 in the mutual fund. Consequently, the shortsale-constrained investor is forced to reduce her exposure to the UMD factor and Columbia Acorn Fund because she cannot use the MKT factor to hedge the overall risk of her portfolio.

To understand whether the difference between the traditional and achievable alphas is related to macroeconomic conditions, we compare the time series of the cross-sectional average traditional and achievable alphas computed on a 36-month rolling window. We find that the difference between the average traditional and achievable alphas widens during periods of financial turmoil such as the the two back-to-back recessions in 1980 and 1981–82, the dot-com bubble of the early 2000's, and the great financial crisis of 2009. Thus, shortsale-constrained investors should use achievable alpha to evaluate mutual-fund performance *particularly* during periods of financial crises.

To study whether investors indeed rely on the achievable alpha to evaluate mutual-fund performance, we examine whether the traditional and achievable alphas explain future fund flows. We find that the traditional and achievable alphas are jointly significant in explaining future fund flows, suggesting that at least some mutual-fund investors rely on the achievable alpha to evaluate fund performance and make investment decisions. Moreover, we find that the predictive power of achievable alpha increases during periods of high market volatility, relative to that of the traditional alpha. This suggests that investors rely on the achievable alpha to evaluate fund performance, particularly during periods of market turmoil.

For robustness, we also evaluate the performance of mutual funds for investors that can engage in a *limited* amount of shortselling. First, we consider an investor who can short *only* the market factor using, for instance, an inverse market ETF. We find that the availability of inverse ETFs that allow investors to efficiently short the market may help to improve the value of active funds for shortsale-constrained investors; however, even in this case, the average achievable alpha is still substantially smaller than the average traditional alpha of an unconstrained investor. Second, we consider the case of an investor who can short the benchmark factors, but faces a leverage constraint. We find that the average achievable alpha of a leverage-constrained investor remains substantially smaller than the traditional alpha of an unconstrained investor even when we allow the investor to hold aggregate short positions up to 40% of her aggregate long positions.

An important implication of our work is that the value of active fund management for shortsale-constrained investors is significantly smaller than previously thought. This implies that investment platforms with retail investor clients *should* report mutual-fund performance not only in terms of the traditional alpha but also in terms of achievable alpha that accounts for the shortsale impediments faced by the majority of retail investors. Similarly, investment firms that manage the assets of pension funds that (by mandate or strategy) abstain from shorting assets should report mutual-fund performance in terms of achievable alpha or value-added to account for their client's constraints. Finally, our work highlights the importance of the choice of benchmark for evaluating mutual-fund performance, in line with the findings of Mullally and Rossi (2024) and Chen, Evans, and Sun (2025).

There is an extensive literature that evaluates mutual-fund performance using the traditional alpha. This research typically shows that the average active fund earns a negative alpha net of fees (Jensen, 1968; Elton, Gruber, and Blake, 1996; Ferreira, Keswani, Miguel, and Ramos, 2013). However, several studies document the existence of a subset of managers that outperform their benchmarks (Wermers, 2000; Barras, Scaillet, and Wermers, 2010; Fama and French, 2010; Kacperczyk, Nieuwerburgh, and Veldkamp, 2014). Assuming there are diseconomies of scale in fund management, Berk and Green (2004) explain that fund net alpha should be zero in equilibrium because investors allocate capital to funds with positive net alpha until diseconomies of scale drive their net alpha to zero. Thus, manager skill should be measured in terms of gross (instead of net) alpha. More recently, Berk and van Binsbergen (2015) propose using the value a mutual fund extracts from capital markets as the appropriate measure of skill, and find that the average value-added of a mutual fund is about $3.2 million per year. They also show that more than 40% of the funds generate a positive value-added. Barras, Gagliardini, and Scaillet (2022) develop a flexible and bias-adjusted approach to examine value-added across individual funds and find that the majority of funds generate a positive value-added. We contribute to this literature by demonstrating that the performance of a mutual fund for shortsale constrained investors is measured by the achievable alpha, and showing that the proportion of funds with positive gross alpha or value-added is much smaller from the perspective of a shortsale-constrained investor.

Our work is also related to the literature that studies the effect of market frictions on the benefits to investors from holding different asset classes. For instance, De Roon, Nijman, and Werker (2001) show that in the presence of transaction costs and shortsale constraints, U.S. investors no longer benefit from investing in emerging markets. Brown, Gonçalves, and Hu (2024) show that illiquidity and underdiversification in private markets reduce the benefits to investors from holding private-capital assets, such as buyout, venture capital, and real estate. We contribute to this literature by examining whether shortsale-constrained investors benefit from holding actively managed mutual funds.

Finally, our work is related to the literature on market frictions in asset pricing (Novy-Marx and Velikov, 2016; Barroso and Detzel, 2021; DeMiguel, Martin-Utrera, Nogales, and Uppal, 2020; Chen and Velikov, 2023; Detzel, Novy-Marx, and Velikov, 2023; Li, DeMiguel, and Martin-Utrera, 2024; DeMiguel, Martin-Utrera, and Uppal, 2024; Muravyev, Pearson, and Pollet, 2025). While this literature focuses on the impact of market frictions on the performance of asset-pricing models, we focus on the impact of market frictions on the performance of mutual funds.

The rest of the paper is organized as follows. In Section 2, we show theoretically how to measure mutual-fund alpha in the presence of shortsale constraints. In Section 3, we first describe our data and methodology for constructing multifactor benchmark portfolios and then discuss our empirical results. Section 5 evaluates the performance of mutual funds for investors that can engage in a *limited* amount of shortselling. Section 6 concludes. Proofs of all our theoretical results are provided in the appendix.

2 Achievable Alpha: Theoretical Results

In this section, we provide our theoretical results that show that one can interpret the traditional and achievable mutual fund alphas as the marginal mean-variance utility improvement in the absence and presence of shortsale constraints of an investor who has access to the fund in addition to the benchmark factors. We also characterize the difference between the traditional and achievable alphas and the conditions under which achievable alpha will be smaller than the traditional alpha. Table 1 summarizes our notation.

Mutual-fund performance is traditionally measured using the fund's alpha (Jensen, 1968), defined as the intercept from regressing the fund's returns on the returns of the benchmark factors. The use of the traditional alpha as a fund performance measure is economically justified by the work of Gibbons et al. (1989), who show that a quadratic form of the alpha equals the increase in the squared Sharpe ratio of a mean-variance investor that has access to the fund in addition to the benchmark factors.

Table 1: Guide to notation

This table describes the notation we use in the paper to describe the excess returns of a mutual fund and benchmark factors, the traditional and achievable alpha, and the slope coefficients (betas) obtained from various regressions of excess returns. The first column of the table gives the symbol we use, and the second column its definition.

Notation	Definition
$R_{mf,t}$	mutual-fund return in excess of the risk-free rate
$R_{b,t}$	excess return of benchmark factors
$R_{b_+,t}$	excess return of benchmark factors with positive weight in the shortsale-constrained mean-variance portfolio
$R_{b_0,t}$	excess return of benchmark factors with zero weight in the shortsale-constrained mean-variance portfolio
$\alpha_{\mathcal{T}}$	traditional alpha, intercept from regressing $R_{mf,t}$ on $R_{b,t}$
$\alpha_{\mathcal{A}}$	achievable alpha, intercept from regressing $R_{mf,t}$ on $R_{b_+,t}$
α_{0+}	intercept from regressing $R_{b_0,t}$ on $R_{b_+,t}$
$\beta_{\mathcal{T}}$	slope from regressing $R_{mf,t}$ on $R_{b,t}$
$\beta_{\mathcal{A}}$	slope from regressing $R_{mf,t}$ on $R_{b_+,t}$
$\beta_{\mathcal{T},+}$	slope coefficient on $R_{b_+,t}$ when regressing $R_{mf,t}$ on $R_{b_+,t}$ and $R_{b_0,t}$
$\beta_{\mathcal{T},0}$	slope coefficient on $R_{b_0,t}$ when regressing $R_{mf,t}$ on $R_{b_+,t}$ and $R_{b_0,t}$

Berk and Green (2004) show that if markets are competitive and there are diseconomies of scale in mutual-fund management, in equilibrium, a fund's gross alpha should be equal to its management fee, and thus, investors should be indifferent between investing in the fund or not. In this context, it is useful to relate a fund's alpha to the *marginal (per dollar)* utility improvement that it can generate for an investor with access to the benchmark factors. This interpretation is highlighted in the following well-known result.

Proposition 1 *A fund's* traditional *alpha, $\alpha_{\mathcal{T}}$, defined as the intercept from regressing the mutual-fund return in excess of the risk-free rate $R_{mf,t}$ on the benchmark factor returns $R_{b,t}$,*

$$R_{mf,t} = \alpha_{\mathcal{T}} + \beta_{\mathcal{T}} R_{b,t} + \epsilon_{b,t}, \tag{1}$$

is the marginal *mean-variance utility improvement of an unconstrained investor who has access to the fund in addition to the benchmark factors.*

The marginal utility improvement measured by the traditional alpha, as shown in Proposition 1, can only be realized if the investor can invest in the optimal mean-variance portfolio of the benchmark factors. However, this portfolio may require taking a short position in some of the benchmark factors and many investors face shortsale impediments in practice. The following proposition shows that the *achievable* alpha, defined as the intercept from regressing the fund returns on the returns of those benchmark factors with a strictly positive weight in the shortsale-constrained mean-variance portfolio, measures the marginal utility improvement that a fund generates for a shortsale-constrained investor.

Proposition 2 *Let the weights of the shortsale-constrained mean-variance portfolio of the benchmark factors be* $w_b^* = (w_{b_+}^*, w_{b_0}^*)$*, where* $w_{b_+}^* > 0$ *and* $w_{b_0}^* = 0$*. Then, a fund's achievable alpha,* $\alpha_{\mathcal{A}}$*, defined as the fund's alpha with respect to the returns of the benchmark factors with positive weight in the shortsale-constrained mean-variance portfolio,* $R_{b_+,t}$*,*

$$R_{mf,t} = \alpha_{\mathcal{A}} + \beta_{\mathcal{A}} R_{b_+,t} + \epsilon_{b_+,t}, \tag{2}$$

is the marginal mean-variance utility improvement that a shortsale-constrained investor can achieve by investing in the fund in addition to the benchmark factors.

The theoretical result in Proposition 2 is closely related to that of De Roon et al. (2001), who propose regression-based results for mean-variance spanning in the case where investors face shortsale constraints and transaction costs. However, in the following proposition we go beyond the analysis in De Roon et al. (2001) to establish also the conditions under which the achievable alpha is smaller than the traditional alpha.

Proposition 3 *Let* $R_{b_+,t} \in R^{K_+}$ *denote the return of the benchmark factors with strictly positive weight and* $R_{b_0,t} \in R^{K_0}$ *the return of the benchmark factors with zero weight in the shortsale-constrained mean-variance portfolio. Regressing the mutual fund excess returns,* $R_{mf,t}$*, on the benchmark factor returns, we have that:*

$$R_{mf,t} = \alpha_{\mathcal{T}} + \beta_{\mathcal{T},+} R_{b_+,t} + \beta_{\mathcal{T},0} R_{b_0,t} + \epsilon_{b,t}, \tag{3}$$

where $\alpha_{\mathcal{T}}$ *is the traditional alpha, and regressing the zero-weight factor returns,* $R_{b_0,t}$*, on the positive-weight factor returns,* $R_{b_+,t}$*, we have that:*

$$R_{b_0,t} = \alpha_{0,+} + \beta_{0,+} R_{b_+,t} + \epsilon_{0,+,t}. \tag{4}$$

Then, the difference between the traditional and achievable alphas is

$$\alpha_{\mathcal{T}} - \alpha_{\mathcal{A}} = -\beta_{\mathcal{T},0}\, \alpha_{0,+}. \tag{5}$$

Moreover, $\alpha_{0,+} < 0$*, and thus, the achievable alpha is smaller than the traditional alpha if the fund has strictly positive exposure to at least one zero-weight factor and nonnegative exposure to the rest.*

Intuitively, one would expect that the achievable alpha is larger than the traditional alpha because the achievable alpha is the abnormal return with respect to a *subset* of the factors used to compute the traditional alpha. However, the above proposition shows that if the fund has a positive exposure to the factors for which the shortsale-constrained mean-variance portfolio assigns a zero weight, then the achievable alpha will actually be *smaller* than the traditional alpha. This is because shorting the zero-weight factors allows the investor to hedge the risk of the mutual-fund returns. In the next section, first, in Section 3.3, we provide a numerical example to illustrate this intuition and then show empirically that the achievable alpha is, on average, smaller than the traditional alpha because, on average, mutual funds have positive exposure to the zero-weight benchmark factors.

3 Achievable Alpha: Empirical Performance

In this section, we present our empirical results. In Section 3.1, we describe the data we use for our analysis. In Section 3.2, we analyze the characteristics of the mean-variance portfolios for the different benchmark factor models. In Section 3.4, we examine the performance of mutual funds in terms of achievable and traditional alpha and value-added.

3.1 Data

Table 2 lists the seven models we consider as benchmarks: the CAPM model of Sharpe (1964), the four-factor model obtained by adding momentum to the three factors of Fama and French (1993) as in Carhart (1997), FFC, the five-factor model of Fama and French (2015), FF5, the six-factor model of Fama and French (2018), FF6, the four-factor model of Hou et al. (2015), HXZ, a five-factor model with the Hou et al. (2015) factors plus momentum, HXZM, and an eight-factor model based on the eight US domestic equity Vanguard funds in Berk and van Binsbergen (2015), VANG.[8] For models that contain long-short factors, we consider two versions, one with original long-short factors and the other with long-only factors, constructed using only the long leg of the original factors.

[8]Of the 11 Vanguard funds Berk and van Binsbergen (2015) consider for their benchmark model, we consider a factor model that contains the returns of the eight funds that invest only in domestic equity. In particular, we consider the following Vanguard funds: VFINX (large-cap blend), VEXMX (mid-cap blend), NAESX (small-cap blend), VVIAX (large-cap value), VBINX (balanced), VIMSX (mid-cap blend), VISGX (small-cap growth), VISVX (small-cap value).

Table 2: List of factor models considered

This table lists the factor models we consider. The first column gives the acronym of the model, the second column the number of factors in the model (K), the third column the authors who proposed the model, and the fourth column the date and journal of publication. The last column lists the acronyms of the factors included in the model.

Acronym	K	Authors	Date, Journal	Factor acronyms
CAPM	1	Sharpe	1964, JF	MKT
FFC	4	Carhart	1997, JF	MKT, SMB, HML, UMD
FF5	5	Fama and French	2015, JFE	MKT, SMB, HML, RMW, CMA
FF6	6	Fama and French	2018, JFE	MKT, SMB, HML, RMW, CMA, UMD
HXZ	4	Hou, Xue, and Zhang	2015, RFS	MKT, ROE, IA, ME
HXZM	5	Hou, Xue, and Zhang	2015, RFS	MKT, ROE, IA, ME, UMD
VANG	8	Berk and van Binsbergen	2015, JFE	VFINX, VEXMX, NAESX, VVIAX, VBINX, VIMSX, VISGX, VISVX

For the mutual fund data, we start with the dataset in Barras et al. (2022) that consists of 2,321 active mutual funds spanning January 1975 through December 2019.[9] Their data consist of monthly net-of-fees returns, expense ratios, and total net assets for open-end actively managed U.S. equity funds from CRSP, which they then use to compute gross-of-fees returns in excess of the one-month treasury bill (risk-free) rate.[10] Just as in Barras et al. (2022), we compute value-added in terms of January 1, 2000 dollars.

3.2 Mean-Variance Portfolio of Benchmark Factors

In Section 2, we showed that the performance of a mutual fund for a shortsale-constrained investor should be measured using the achievable alpha, defined as the alpha of the mutual fund with respect to only those factors in the benchmark model that have a strictly positive weight in the mean-variance portfolio. To see if shortsale constraints are important for mutual-fund performance, we first compare the weights of the unconstrained and shortsale-constrained mean-variance portfolios for each of the seven benchmark factor models. If the unconstrained and shortsale-constrained portfolios are identical, then the traditional and achievable alphas will coincide.

Figure 2 depicts the weights of the mean-variance portfolio of the factors for each of the seven models listed in Table 2. Panels A and B of this figure depict the *unconstrained* portfolio weights for models based on long-short and long-only factors, respectively. Panel C

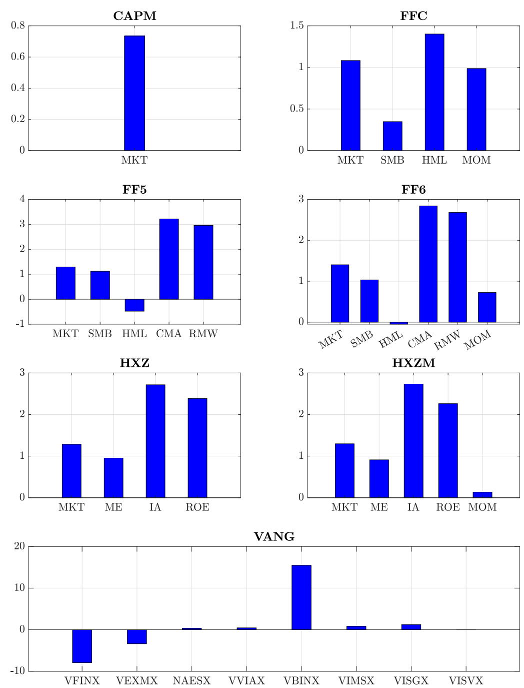
[9] We thank Laurent Barras for providing this data.

[10] The online appendix of Barras et al. (2022, sec. V.A) explains the filters used to construct this dataset.

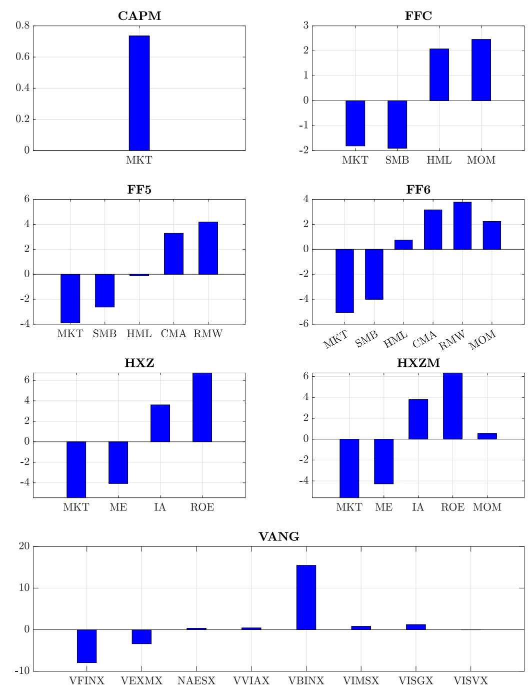
Figure 2: Mean-variance portfolio of benchmark factors

This figure depicts the weights of the mean-variance portfolio of the factors for each of the seven models listed in Table 2. Panels A and B depict the unconstrained portfolio weights for models based on long-short and long-only factors, respectively. Panel C depicts the shortsale-constrained portfolio weights for the model based on long-only factors. We consider an investor with risk aversion parameter $\gamma = 5$ and construct shortsale-constrained mean-variance portfolios for our sample from January 1975 to December 2019.

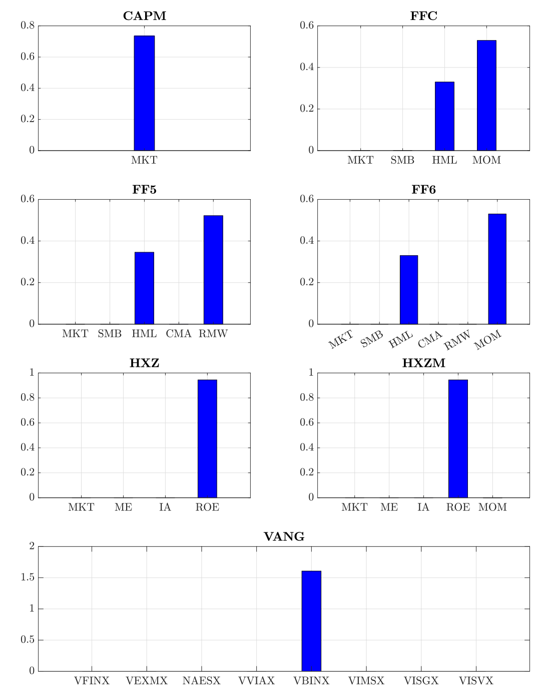
Panel A: Unconstrained long-short factors



Panel B: Unconstrained long-only factors



Panel C: Constrained long-only factors



depicts the *shortsale-constrained* portfolio weights for models based on the long-only factors. We consider an investor with risk aversion parameter $\gamma = 5$ and compute the unconstrained mean-variance portfolios for our entire sample from January 1975 to December 2019.

Panel A of Figure 2 shows that the unconstrained mean-variance portfolio weights for the *long-short* version of the FFC, FF5, FF6, HXZ, and HXZM models do not include substantially negative weights. In particular, the portfolio weights for the FFC, HXZ, and HXZM models are all strictly positive, and those for the FF5 and FF6 models include a single negative weight for the value factor (HML), which is also small in magnitude compared to the other portfolio weights. In contrast, Panel B shows that the unconstrained mean-variance portfolios for the *long-only* version of the factor models contain significant negative weights. For instance, the portfolio weights for the MKT and SMB factors are negative for the FFC, FF5, and FF6 models, and those for the MKT and ME factors are negative for the HXZ and HMXZ models. Panel C depicts the weights of the *shortsale-constrained* mean-variance portfolio of the long-only factors. Comparing the unconstrained and shortsale-constrained portfolios of long-only factors in Panels B and C of Figure 2, we note that the shortsale-constrained portfolio weights are positive for only a few of the long-only factors for each model. For instance, we see from Panel C that for FFC and FF6, only the HML and MOM weights are positive; for FF5, only the HML and RMW weights are positive; and for HXZ and HXZM, only the weight on ROE is positive.

To understand the difference between the unconstrained mean-variance portfolio weights of the long-short and long-only factor models, we report in Table 3 the time-series correlation for the factors of the CAPM, FFC, FF5, FF6, HXZ, and HXZM models described in Table 2. Panel A reports the correlation for the long-short factors considered in the original version of these models, and Panel B for the long-only version of the factors. We see from Panel A that the long-short factor correlations are generally small, which explains why the mean-variance investor tends to assign a positive weight to each long-short factor. In contrast, Panel B shows that the long-only factors tend to be positively correlated to each other, and thus, the mean-variance investor goes long some factors and short others to diversify the portfolio risk.[11] Note also that the original CAPM and VANG models contain

[11] Green and Hollifield (1992) explain that the mean-variance portfolio weight is likely to contain substantial long and short positions if asset returns are driven by a common factor. The long-only factors for each of

Table 3: Factor correlations

This table reports time-series correlation for the factors of the CAPM, FFC, FF5, FF6, HXZ, and HXZM models described in Table 2. Panel A reports the correlation for the long-short factors considered in the original version of these models, and Panel B for the long-only version of the factors. In both panels, we also include the market (MKT), which is a long-only factor.

	SMB	HML	CMA	RMW	UMD	ME	IA	ROE
Panel A: Long-short factor correlations								
MKT	0.27	-0.29	-0.36	-0.27	-0.15	0.25	-0.34	-0.24
SMB		-0.23	-0.10	-0.48	0.02	0.95	-0.19	-0.40
HML			0.67	0.27	-0.18	-0.07	0.70	-0.00
CMA				0.01	-0.01	-0.03	0.91	-0.09
RMW					0.10	-0.39	0.17	0.68
UMD						0.03	0.00	0.51
ME							-0.08	-0.30
IA								0.08
Panel B: Long-only factor correlations								
MKT	0.89	0.89	0.94	0.96	0.92	0.88	0.93	0.96
SMB		0.92	0.97	0.94	0.93	1.00	0.97	0.94
HML			0.95	0.93	0.85	0.91	0.94	0.89
CMA				0.95	0.92	0.96	0.99	0.95
RMW					0.93	0.93	0.95	0.98
UMD						0.92	0.92	0.96
ME							0.97	0.94
IA								0.96

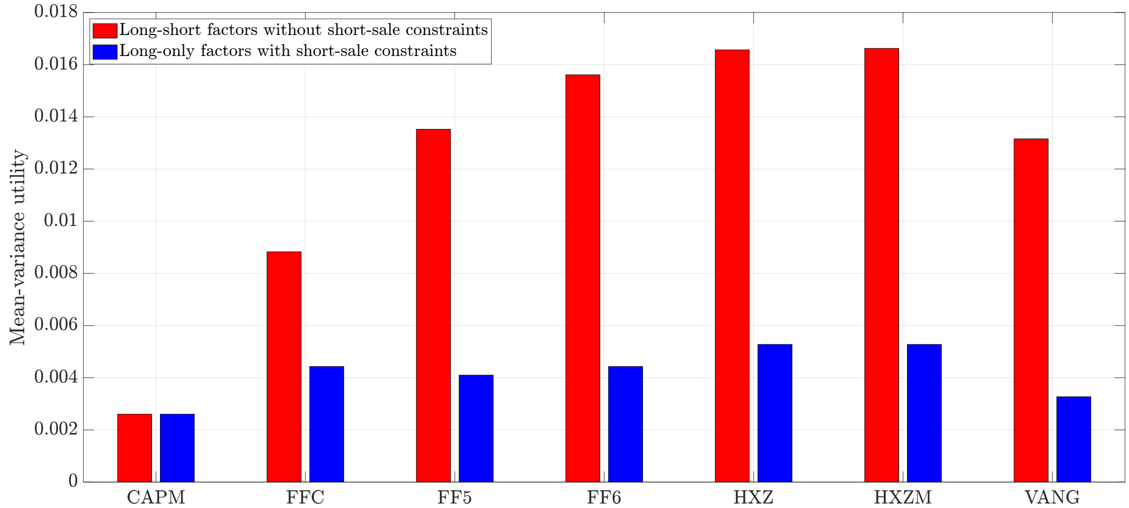
just long-only factors, so the portfolio weights for these models are identical in Panels A and B of Figure 2.

As mentioned above, the CAPM and VANG models are originally composed of long-only factors. Comparing the unconstrained (Panel A or B) and shortsale-constrained (Panel C) mean-variance portfolio weights of the CAPM and VANG models in Figure 2, we observe that, as one would expect, the CAPM model is unaffected by shortsale constraints because the investor optimally assigns a strictly positive weight to the MKT factor even in the absence of shortsale constraints. However, the mean-variance portfolio of the VANG factor model changes dramatically with the presence of shortsale constraints. While in the absence of shortsale constraints (Panels A or B), the VANG portfolio contains substantial negative weights in the VFINX and VEXMX funds and positive weights on five separate funds, in the presence of shortsale constraints (Panel C), it contains a single positive weight

the six models are all likely to be exposed to the market factor, and thus, it is not surprising that the unconstrained mean-variance portfolio of the long-only benchmark factors includes substantial negative and positive weights.

Figure 3: Mean-variance utility of factor models

This figure depicts the mean-variance utility of the different factor models for two cases: (i) long-short factors without shortsale constraints and (ii) long-only factors with shortsale constraints.



on the VBINX fund. This is because, being long-only portfolio returns, the VANG factors are highly correlated to each other.

Figure 3 depicts the mean-variance utility of the different factor models for the following two cases: (1) long-short factors without shortsale constraints and (2) long-only factors with shortsale constraints. The main observation from Figure 3 is that the mean-variance utility of long-only factor models in the presence of shortsale constraints (blue bars) is less than half that of long-short models in the absence of shortsale constraints (red bars) for every model except CAPM. A second observation from Figure 3 is that, while the performance of the CAPM is not affected by short-sale constraints, its performance compares poorly to that of the other factor models—both in the absence and presence of shortsale constraints—and so it is a weak benchmark.

3.3 A simple example to illustrate the key intuition

As mentioned in Section 2, intuitively, one would expect the achievable alpha to be larger than the traditional alpha because the achievable alpha is the abnormal return with respect to a *subset* of the factors used to compute the traditional alpha. However, Proposition 3 shows that if the fund is positively exposed to the factors for which the shortsale-constrained

Table 4: A simple example

This table reports various statistics for Columbia Acorn Fund with respect to a simple benchmark model with only two factors: the market (MKT) and the long-only version of momentum (UMD). The first column reports whether the statistics are for the unconstrained portfolio (traditional alpha) or shortsale-constrained portfolio (achievable alpha). Columns (2)–(5) report the utility and portfolio weights of the mean-variance portfolio of the two factors and the mutual fund. Columns (6)–(8) report the alpha and the beta of the mutual fund with respect to the two factors. We consider an investor with relative risk aversion $\gamma = 5$. All quantities are obtained using the returns of the two factors and the mutual fund for the entire sample from January 1975 to December 2019.

	Mean-variance portfolio				Regression		
	Utility	w_{MKT}	w_{UMD}	w_{mf}	α	β_{MKT}	β_{UMD}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Unconstrained: Traditional	0.0054	−1.15	0.91	0.89	0.0179	0.39	0.56
Shortsale-constrained: Achievable	0.0045	0	0.33	0.53	0.0121	—	0.86

mean-variance portfolio assigns a zero weight, then the achievable alpha will actually be *smaller* than the traditional alpha.

To explain the intuition for this result, we consider a simple example with only two factors: the market (MKT) and the long-only version of momentum (UMD). Table 4 reports various statistics for Columbia Acorn Fund with respect to this simple benchmark model with only these two factors. The first column reports whether the statistics are for the unconstrained portfolio (traditional alpha) or the shortsale-constrained portfolio (achievable alpha). Columns (2)–(5) report the utility and portfolio weights of the mean-variance portfolio of the two factors and the mutual fund. Columns (6)–(8) report the alpha and betas of the mutual fund with respect to the two factors. We consider an investor with relative risk aversion $\gamma = 5$. All quantities are obtained using the returns of the two factors and the mutual fund for the entire sample from January 1975 to December 2019.

Consistent with the results in Figure 2, Table 4 shows that the unconstrained mean-variance portfolio of these two factors assigns a weight of −0.80 to MKT and 1.40 to UMD, but the shortsale-constrained mean-variance portfolio assigns a weight of zero to MKT and 0.79 to UMD. The reason the unconstrained investor finds it optimal to go long the momentum factor and short the market is because, as discussed in Section 3.2, the correlation between the MKT and the long-only version of UMD is 92%; moreover, the monthly mean return of long-only UMD is 1.08%, whereas that of MKT is only 0.71%.

We then compute the traditional and achievable monthly alphas of Columbia Acorn Fund with respect to this two-factor model and find that they are 1.79% and 1.21%.[12] Thus, the achievable alpha is substantially smaller than the traditional alpha. This is consistent with Proposition 3 because Columbia Acorn Fund has positive betas of 0.39 and 0.56 with respect to the MKT and long-only UMD factors.

To understand why the achievable alpha is smaller than the traditional alpha, we compare the unconstrained and shortsale-constrained mean-variance portfolio weights of the two benchmark factors and the mutual fund. We find that an unconstrained investor would hold -1.15 in MKT, 0.91 in UMD, and 0.89 in the mutual fund. That is, the unconstrained investor would short the MKT to hedge the risk of the UMD factor and the mutual fund, achieving an overall mean-variance utility of 0.0054. In contrast, a shortsale-constrained investor would hold zero in MKT, 0.33 in UMD, and only 0.53 in the mutual fund. That is, the shortsale-constrained investor is forced to reduce her exposure to the UMD factor and Columbia Acorn Fund because she cannot use the MKT factor to hedge the overall risk of her portfolio and, as a result, achieves a smaller mean-variance utility of only 0.0045.

Consistent with the results for the motivating example reported in the sixth column of Table 4, in Section 3.4, we find empirically that the achievable alpha is, on average, smaller than the traditional alpha.

3.4 Mutual-Fund Performance

In this section, we discuss mutual-fund performance in terms of alpha and value-added. We also examine whether shortsale constraints change the rankings of funds.

3.4.1 Achievable alpha

Table 5 reports cross-sectional statistics for the traditional and achievable fund gross alphas with respect to the seven factor models listed in Table 2. Panel A reports cross-sectional statistics for the traditional alpha with respect to the long-short factors, obtained by regressing the fund returns on all long-short factor for each model, and Panel B for the achievable

[12]The returns of the MKT, long-only version of UMD, and Columbia Acorn Fund are available for our entire sample from January 1975 to December 2019. Thus, all numbers we report are obtained using the entire sample of data.

Table 5: Traditional and achievable mutual-fund alphas

This table reports cross-sectional statistics for the traditional and achievable fund gross alphas with respect to the seven factor models listed in Table 2. Panel A reports cross-sectional statistics for the traditional alpha with respect to the long-short factors, obtained by regressing the fund returns on all long-short factor for each model, and Panel B for the achievable alpha with respect to the long-only factors, obtained by regressing fund returns on just those long-only factors with a strictly positive weight in the shortsale-constrained mean-variance portfolio (over the sample period for which we have return data for the fund). We report the average alpha across funds, its t-statistic, the time-weighted average alpha (where the weight is proportional to the length of the sample period for which we have return data for the fund), and its t-statistic. We also report percentiles of the cross-sectional distribution of fund alpha and the percentage of funds with positive alpha and t-statistic greater than two. Alphas are annualized and reported in percentage. Like Barras et al. (2022), we winsorize observations that are more than five times the inter-decile range (difference between the 90th and 10th percentiles) away from the median.

	CAPM	FFC	FF5	FF6	HXZ	HXZM	VANG
Panel A: Traditional alpha with respect to long-short factors							
Average alpha	0.61	0.31	0.50	0.42	0.17	0.24	0.58
t-stat	11.68	6.86	8.67	7.95	3.01	4.66	11.99
Time-weighted average alpha	0.87	0.58	0.63	0.55	0.38	0.42	0.98
t-stat	4.28	4.20	4.12	4.11	3.78	3.94	4.30
10th percentile	-2.23	-2.10	-2.45	-2.23	-2.71	-2.50	-2.07
50th percentile	0.59	0.31	0.24	0.18	0.06	0.08	0.54
90th percentile	3.33	2.75	3.85	3.37	3.39	3.28	3.25
Percentage of funds with $\alpha > 0$	62.76	57.37	53.92	54.05	51.21	51.77	62.54
Percentage of funds with $t(\alpha) >2$	7.89	9.14	12.76	11.12	8.62	9.48	14.91
Panel B: Achievable alpha with respect to long-only factors							
Average alpha	0.58	-1.79	-2.05	-2.34	-3.28	-3.29	-1.14
t-stat	11.18	-23.97	-30.61	-32.82	-44.38	-44.47	-16.73
Time-weighted average alpha	0.86	-1.17	-1.44	-1.71	-2.75	-2.75	-0.31
t-stat	4.27	-4.27	-4.32	-4.33	-4.36	-4.36	-3.42
10th percentile	-2.24	-5.87	-5.40	-6.12	-7.15	-7.09	-4.57
50th percentile	0.57	-1.17	-1.56	-1.76	-2.67	-2.68	-0.91
90th percentile	3.30	1.78	1.09	0.99	0.26	0.25	2.25
Percentage of funds with $\alpha > 0$	62.54	30.81	22.32	20.85	11.81	11.72	37.27
Percentage of funds with $t(\alpha) > 2$	7.76	3.02	1.29	1.29	0.30	0.34	4.44

alpha with respect to the long-only factors, obtained by regressing fund returns on just those long-only factors with a strictly positive weight in the shortsale-constrained mean-variance portfolio (over the sample period for which we have return data for the fund). We report the average alpha across funds, its t-statistic, the time-weighted average alpha (where the weight is proportional to the length of the sample period for which we have return data for the fund), and its t-statistic. We also report percentiles of the cross-sectional distribution of fund alpha and the percentage of funds with positive alpha and t-statistic greater than two. Alphas are annualized and reported in percentage. Like Barras et al. (2022), we winsorize observations that are more than five times the inter-decile range (difference between the 90th and 10th percentiles) away from the median.

Comparing Panels A and B of Table 5, we find that mutual-fund performance measured in terms of the achievable alpha (of a shortsale-constrained investor) with respect to the long-only versions of the factor models is substantially worse than that measured in terms of traditional alpha with respect to the long-short factor models. For instance, while the proportion of mutual funds with positive traditional alpha with respect to the long-short factor models ranges from 51.21% for HXZ to 62.54% for VANG, the proportion of mutual funds with positive achievable alpha with respect to the long-only models is 11.81% for HXZ and 37.27% for VANG.[13] This finding is robust to evaluating mutual-fund performance in terms of alpha t-statistic. Comparing the last row in Panels A and B of Table 5, we find that while the proportion of mutual funds with significant ($t(\alpha) > 2$) traditional alpha with respect to the long-short factor models ranges from 8.62% for HXZ to 14.91% for VANG, the proportion of mutual funds with significant achievable alpha with respect to the long-only models is 0.30% for HXZ and 4.44% for VANG.

Thus the main takeaway is that considering jointly long-only benchmark factors and shortsale constraints leads to a substantial deterioration of the performance of the cross-section of mutual funds. This is because the mean-variance portfolio of the long-only factors contains substantial negative positions on the factors, unlike that for the long-short factors, as shown in Panels A and B of Figure 2. The explanation is that the long-only factors are highly correlated with each other and, thus, an *unconstrained* mean-variance investor would optimally short some of the factors to manage the portfolio risk.

[13]The CAPM is the only benchmark relative to which mutual-fund performance using the traditional alpha is similar to that using the achievable alpha. However, as shown in Figure 3, the CAPM is a weak benchmark compared to the other factor models. Observe also that the traditional and achievable alphas [and value-added] with respect to the CAPM in Panels A and B of Table 5 [and Table 6] are small but not zero. To understand the reason for this, note that we compute the alpha for each fund separately over the entire subsample of months for which we have return data for the fund. Although for our entire sample, it is optimal to long the market as shown in Panels A and B of Figure 2, for the specific subsamples for which we have data for some of the mutual funds, it is optimal to short the market. Therefore, for those specific mutual funds, the achievable and traditional alpha are different. This is illustrated in the first subfigure in Figure 4, which shows that the weight on the MKT for the CAPM model is zero for a very small percentage of mutual funds (the red bar is around 1%).

3.4.2 Achievable value-added

Berk and van Binsbergen (2015) explain the importance of measuring mutual-fund performance in terms of value-added, which they define as the average of the product between a fund's gross abnormal returns and its assets under management. Table 6 reports cross-sectional statistics for the traditional and achievable fund value-added (in January 2000 million dollars) with respect to the seven factor models listed in Table 2. For each fund, we compute the traditional value-added as the average of the product of assets under management and abnormal returns, obtained by regressing the fund returns on all factors in each model, and the achievable value-added computed using the fund abnormal returns with respect to only those factors with a strictly positive weight in the shortsale-constrained mean-variance portfolio. We report the average value-added, its t-statistic, the time-weighted average value-added (where the weight is proportional to the length of the sample period for which we have return data for the fund), and its t-statistic. We also report percentiles of the cross-sectional distribution of value-added and the percentage of funds with positive average value-added and t-statistic greater than two. Value added is annualized.

Similar to Table 5 for traditional and achievable alphas, Table 6 contains two panels: Panel A reports the average traditional value-added for the long-short factor models, and Panel B the average achievable value-added for the long-only factor models. Consistent with the findings of Berk and van Binsbergen (2015), Panel A of Table 6 shows that the average *traditional* value-added in the cross-section of mutual funds is generally negative when computed with respect to conventional factor models, ranging from just 0.01 million dollars for the FF5 model to -1.71 million dollars for the CAPM model. We also confirm the result of Berk and van Binsbergen (2015) that the cross-sectional average value-added is significantly positive with respect to the VANG model at 0.78 million dollars, with a t-statistic of 2.36.[14]

The key takeaway from Table 6 is that the main findings from evaluating mutual-fund performance in terms of achievable alpha and achievable value-added are similar. For instance, comparing Panels A and B in Table 6, we find that while the proportion of mu-

[14] Note that we consider only funds investing in US equities, whereas Berk and van Binsbergen (2015) consider funds investing in all equities, that is, including international equities. As a result, the value-added they estimate is slightly larger than our estimate. In table 3 of their internet appendix, they show that the value-added decreases when considering funds investing in only US equities.

Table 6: Traditional and achievable mutual-fund value-added

This table reports cross-sectional statistics for the traditional and achievable fund value-added with respect to the seven factor models listed in Table 2. Panel A reports cross-sectional statistics for the traditional value-added computed as the average of the product of assets under management and abnormal returns, obtained by regressing the fund returns on all long-short factors in each model, and Panel B the achievable value-added computed using the fund abnormal returns with respect to just those long-only factors with a strictly positive weight in the shortsale-constrained mean-variance portfolio (over the sample period for which we have return data for the fund). We report the average value-added, its t-statistic, the time-weighted average value-added (where the weight is proportional to the length of the sample period for which we have return data for the fund), and its t-statistic. We also report percentiles of the cross-sectional distribution of value-added and the percentage of funds with positive average value-added and t-statistic greater than two. Value added is annualized and expressed in January 2000 million dollars. Like Barras et al. (2022), we winsorize observations that are more than five times the inter-decile range away from the median.

	CAPM	FFC	FF5	FF6	HXZ	HXZM	VANG
Panel A: Traditional value-added with respect to long-short factors							
Average value-added	-1.71	-0.46	0.01	-0.14	-1.57	-1.21	0.78
t-stat	-4.85	-1.50	0.02	-0.40	-4.07	-3.44	2.36
Time-weighted average value-added	-0.87	0.12	0.20	-0.04	-1.44	-1.00	1.95
t-stat	-1.90	0.33	0.42	-0.09	-2.58	-2.14	3.34
10th percentile	-13.92	-10.26	-11.93	-10.72	-14.16	-12.73	-8.67
50th percentile	-0.81	-0.61	-0.58	-0.62	-0.92	-0.83	-0.30
90th percentile	8.15	7.57	12.17	10.00	8.68	8.43	11.74
% of funds with average value-added > 0	38.37	39.24	41.20	39.66	35.12	35.88	44.58
Panel B: Achievable value-added with respect to long-only factors							
Average value-added	-1.81	-9.56	-12.30	-12.81	-19.18	-19.11	-8.28
t-stat	-5.06	-15.63	-19.45	-20.34	-23.52	-23.52	-18.16
Time-weighted average value-added	-0.97	-10.67	-13.91	-14.90	-23.70	-23.64	-7.15
t-stat	-2.05	-4.28	-4.33	-4.35	-4.37	-4.37	-4.23
10th percentile	-14.09	-32.81	-36.95	-38.01	-51.04	-50.81	-27.62
50th percentile	-0.83	-3.81	-5.11	-5.32	-7.73	-7.74	-3.44
90th percentile	8.14	4.08	1.14	1.23	-0.28	-0.31	2.49
% of funds with average value-added > 0	38.13	20.13	14.30	14.22	8.90	8.90	18.38

tual funds with positive traditional value-added with respect to the long-short factor models ranges from 35.12% for HXZ to 44.58% for VANG, the proportion of mutual funds with positive achievable value-added with respect to the long-only models is 8.90% for HXZ and 18.38% for VANG. In contrast to the findings in Panel A of Table 6 for traditional alpha, Panel B shows that the cross-sectional average *achievable* value-added is significantly negative with respect to every factor model, including VANG. For instance, the cross-sectional average achievable value-added ranges from -1.81 million dollars for CAPM to -19.11 million dollars for HXZM. For the VANG model, the cross-sectional average achievable value-added is −8.28 million dollars, with a t-statistic of −18.16.

3.4.3 Rankings

In the previous sections, we have shown that, on average, mutual-fund performance deteriorates significantly for a shortsale-constrained investor. In this section, we examine whether the *relative* performance of different mutual funds changes with shortsale constraints. To do this, we compare the rankings of funds in terms of achievable versus traditional alphas and value-added.

We measure the difference between the traditional and achievable rankings in terms of their assignment of funds to deciles. Let $T = \{T_1, T_2, \ldots, T_N\}$ and $A = \{A_1, A_2, \ldots, A_N\}$ be the traditional and achievable mutual-fund rankings in terms of traditional and achievable alpha (or value-added). We assign mutual funds to deciles based on their rankings in T and A. The decile for the nth mutual fund in terms of the traditional and achievable rankings can be computed as:

$$d_T(n) = \left\lceil \frac{10 \cdot T_n}{N} \right\rceil \quad \text{and} \quad d_A(n) = \left\lceil \frac{10 \cdot A_n}{N} \right\rceil, \tag{6}$$

where T_n and A_n are the traditional and achievable rankings of the nth mutual fund, N is the total number of mutual funds, and $\lceil \cdot \rceil$ is the ceiling function.

To measure the difference between the deciles constructed according to the traditional and achievable rankings, we count the number of mutual funds that are assigned to different deciles in rankings T and A.

$$\text{Diff} = \frac{1}{N} \sum_{n \in N} \delta(d_T(n) \neq d_A(n)), \tag{7}$$

where $\delta(\cdot)$ is an indicator function equal to one if the condition is true and zero otherwise. The value of "Diff" ranges between zero and one, with zero indicating that every mutual fund is assigned to the same decile in both rankings and one indicating that every mutual fund is assigned to a different decile in the two rankings.

Table 7 reports the difference between the rankings of mutual funds based on the traditional and achievable alpha and value-added, measured using Diff as defined in Equation (7). Panel A reports the ranking difference in terms of traditional and achievable alphas,

Table 7: Difference between traditional and achievable rankings

This table reports the difference between the rankings of mutual funds based on the traditional and achievable alpha and value-added with respect to the long-only version of the seven factor models listed in Table 2. Panel A reports the ranking difference ("Diff") in terms of traditional and achievable alphas, and Panel B in terms of traditional and achievable value-added. For each factor model, we report the measure Diff defined in Equation (7) in percentage.

	CAPM	FFC	FF5	FF6	HXZ	HXZM	VANG
Panel A: Difference in rankings based on alpha							
Diff (%)	3.19	75.99	82.80	81.29	83.66	83.15	73.02
Panel B: Difference in rankings based on value-added							
Diff (%)	1.83	77.22	83.55	82.23	82.82	82.97	78.70

and Panel B in terms of traditional and achievable value-added. Panels A and B show that fund rankings change decile for more than 70% of the funds across every factor model except CAPM. This demonstrates that relative mutual-fund performance is very different from the perspective of a shortsale-constrained investor. This has implications for capital flows, which have been shown to be driven by relative mutual-fund performance (Sirri and Tufano, 1998).

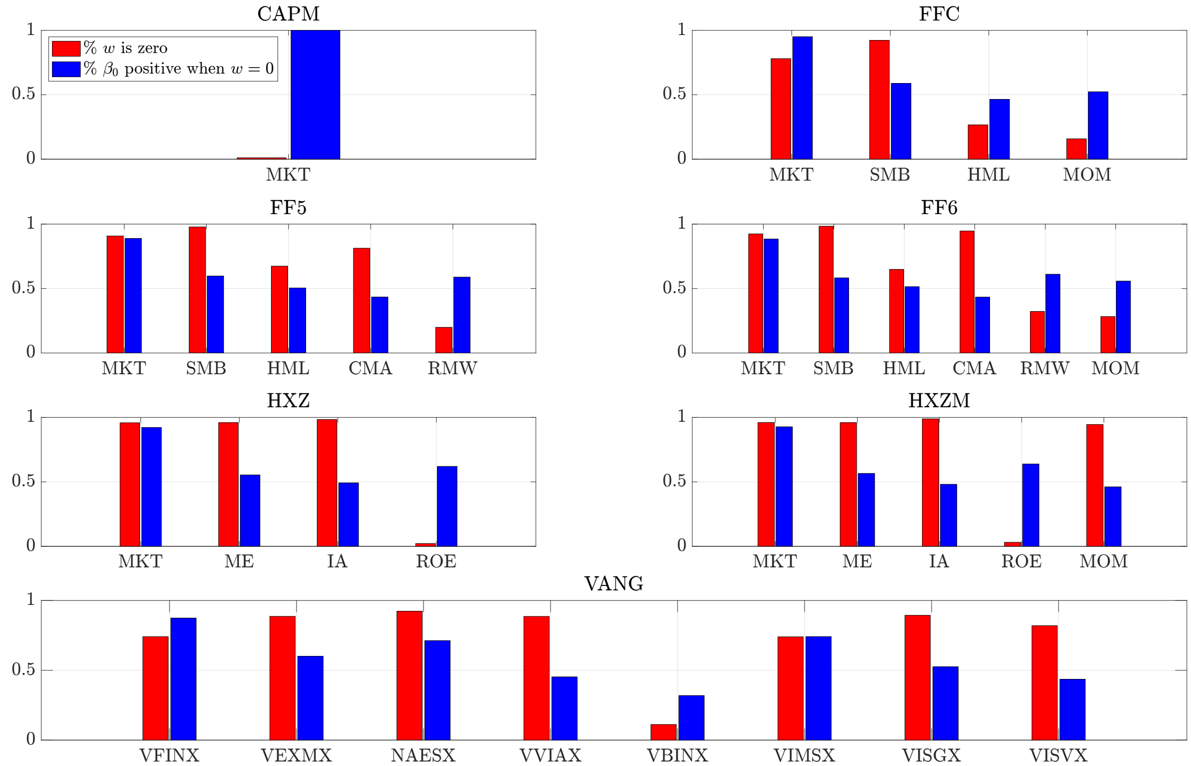
3.4.4 Discussion: Why is achievable performance worse?

Our results consistently show that the achievable alpha and value-added are smaller than their traditional counterparts for the long-only version of the factor models. This is a counterintuitive finding because we compute the achievable alphas by dropping those factors with zero weight from the benchmark in the shortsale-constrained mean-variance portfolio, and one would expect that dropping factors from the benchmark would lead to higher estimated abnormal returns. However, Proposition 3 shows that if a fund has positive exposure to the factors with a zero weight in the shortsale constrained mean-variance portfolio, then the achievable alpha is smaller than the traditional alpha. In this section, we show that the benchmark factors often have a zero weight in the mean-variance portfolio and that mutual funds often have a positive exposure to the factors with zero weight in the mean-variance portfolio.

Figure 4 depicts several statistics for the regression of fund returns on the returns of the long-only benchmark factor models. Each panel reports the results for the long-only version of the seven models in Table 2. For each factor in each panel, we report the proportion of funds for which the factor has a zero weight in the shortsale-constrained mean-variance

Figure 4: Statistics from regression of fund returns on long-only factors

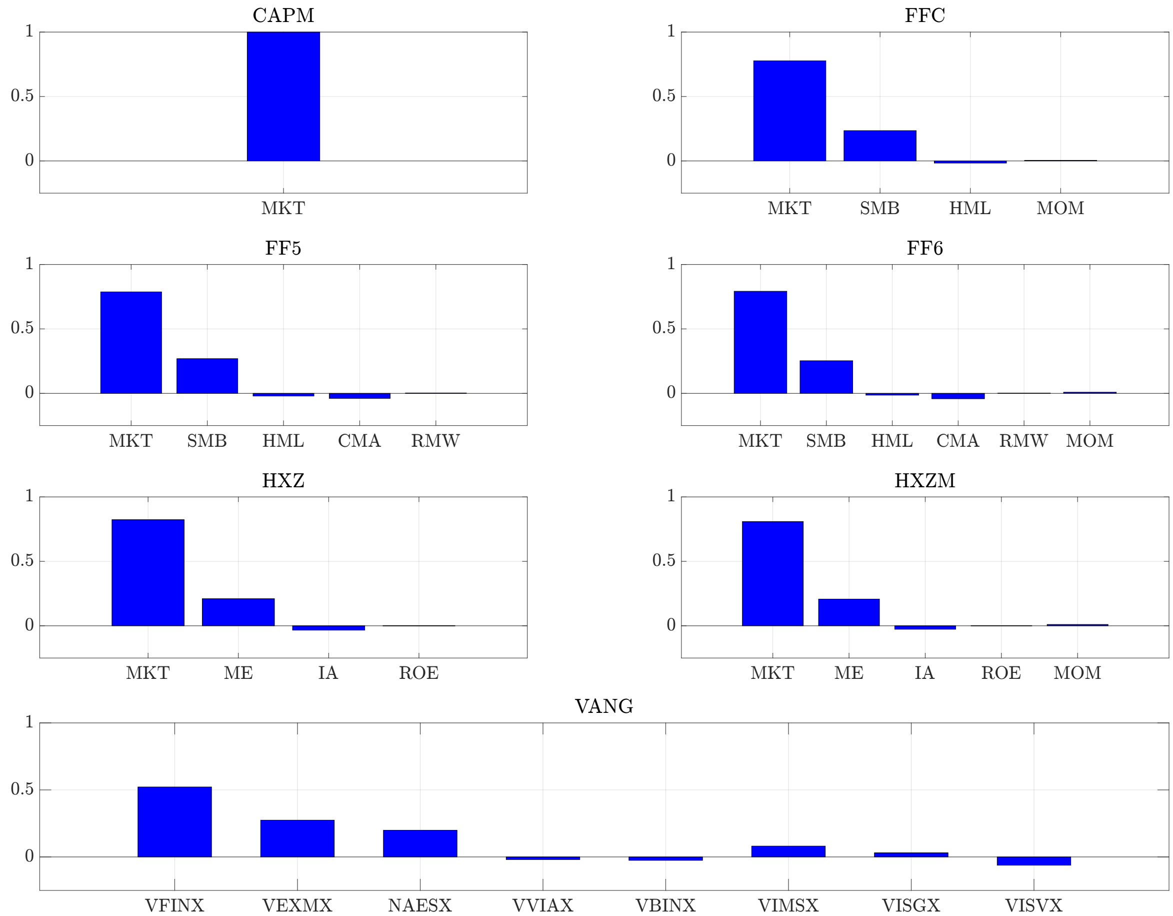
This figure depicts several statistics for the regression of fund returns on the returns of the long-only benchmark factor models. Each panel reports the results for the long-only version of the seven models in Table 2. For each factor in each panel, we report the proportion of funds for which the factor has a zero weight in the shortsale-constrained mean-variance portfolio of the benchmark factors for the sample period for which we have return data for the fund (red bars). We also report the proportion of funds for which the loading on a factor that the shortsale-constrained mean-variance portfolio assigns a zero weight is positive (blue bars). The legend for the figure is displayed in the first plot (for CAPM).



portfolio of the benchmark factors for the sample period for which we have return data for the fund (red bars). We also report the proportion of funds for which the loading on a factor that the shortsale-constrained mean-variance portfolio assigns a zero weight is positive (blue bars). The red bars show that, except for the CAPM, many factors often have a zero weight in the mean-variance portfolio. For instance, for FFC, both MKT and SMB have zero weight in the mean-variance portfolio for more than 50% of the mutual funds. Similarly, for HXZM, we observe that MKT, ME, IA, and MOM have zero weight for almost 100% of the funds. For VANG, we also observe that every factor except VBINX has zero weight in the mean-variance portfolio for more than 50% of the mutual funds. In addition, the blue bars show that many mutual funds have positive exposure to factors with zero weight in the

Figure 5: Contribution of each factor to alpha deterioration

This figure depicts the contribution of each factor to the difference between the traditional and achievable alphas averaged across all funds using Equation (8). Each panel reports the results for the long-only version of the seven models in Table 2.



mean-variance portfolio. For instance, for FFC, FF5, and FF6, we observe that conditional on the MKT and SMB factors having a zero weight in the mean-variance portfolio for the sample period for which we have data for a fund, more than 70% of the funds have positive exposure to MKT and SMB.

Figure 5 reports the contribution of the kth factor to the difference between the traditional and achievable alphas averaged across all N funds. We compute the contribution of each factor using the following expression based on Equation (5) of Proposition 3:

$$\text{Contribution of } k\text{th factor} = \frac{\sum_{n=1}^{N}(-\beta_{\mathcal{T},0,k}\alpha_{0,+,k})}{\sum_{n=1}^{N}(-\beta_{\mathcal{T},0}\alpha_{0,+})} = \frac{\sum_{n=1}^{N}(-\beta_{\mathcal{T},0,k}\alpha_{0,+,k})}{\sum_{n=1}^{N}\sum_{k}^{K_0}(-\beta_{\mathcal{T},0,k}\alpha_{0,+,k})}, \quad (8)$$

where $\beta_{\mathcal{T},0}$ and $\alpha_{0,+}$ are as defined in Proposition 3 and their kth elements are denoted as $\beta_{\mathcal{T},0,k}$ and $\alpha_{0,+,k}$. Figure 5 shows that the market and size factors contribute most to the difference between the traditional and achievable alphas. The intuition for this is that they not only often have a zero weight in the mean-variance portfolio and the mutual funds have a positive beta on them, but also that they have a large negative alpha with respect to the other factors in the model. In particular, note that the factors with the largest contribution to the difference in traditional and achievable alpha are the MKT and SMB factors for the FFC, FF5, and FF6 models, the MKT and ME for the HXZ and HXZM models, and the VFINX (large-cap blend), VEXMX (mid-cap blend), NAESX (small-cap blend) factors for the VANG model.

Taken together, Proposition 3 and Figures 4 and 5 explain why achievable alpha and value-added tend to be smaller than their traditional counterparts in our sample.

4 Economic Interpretation

In this section, we provide economic interpretation for the empirical performance of the achievable alpha. In Section 4.1, we study whether the difference between the traditional and achievable alphas is related to macroeconomic conditions, and in Section 4.2, we study whether past performance (measured by traditional or achievable alpha) predicts mutual-fund flows.

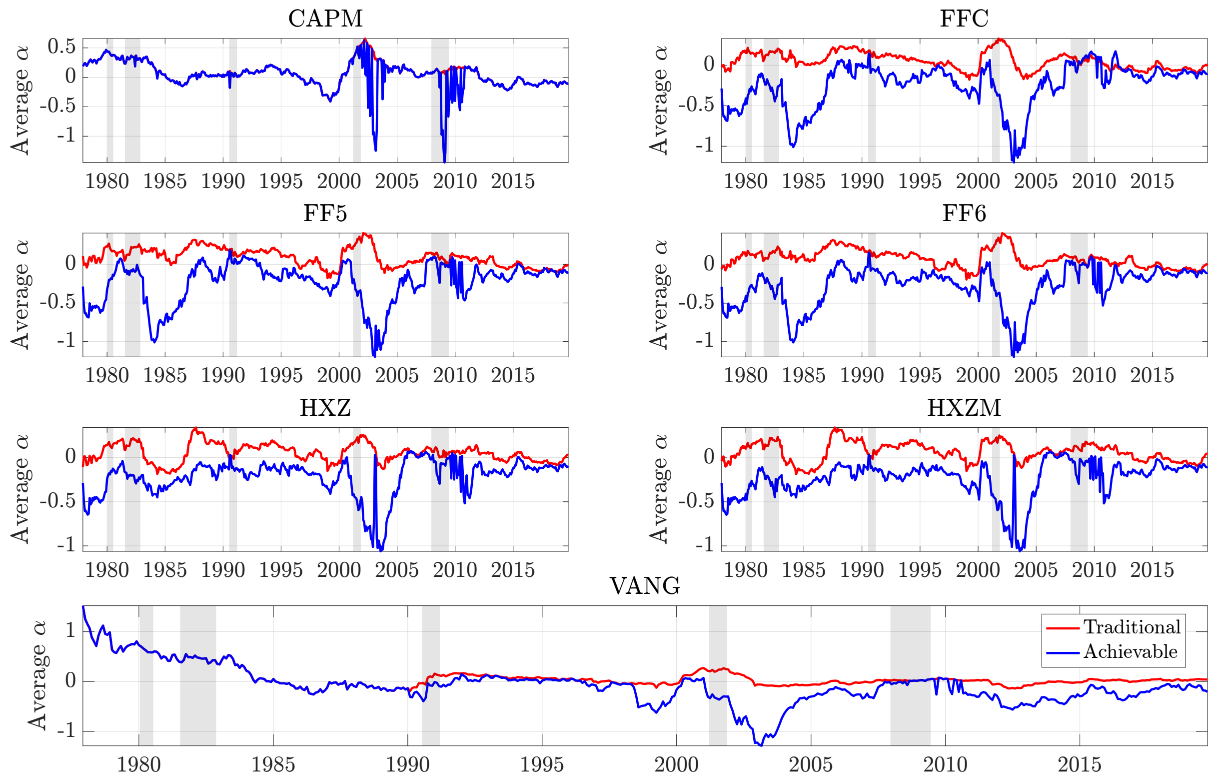
4.1 Time Series of Traditional and Achievable Alpha

To understand whether the difference between the traditional and achievable alphas is related to macroeconomic conditions, Figure 6 depicts the time series of the cross-sectional average traditional and achievable alpha computed on a 36-month rolling window for the seven models in Table 2. Our choice of a 36-month rolling window is motivated by Morningstar and other investment platforms often reporting mutual-fund performance over the past three years. The traditional alpha is computed with respect to all long-short factors for each model and the achievable alpha with respect to just those long-only factors that have a positive weight in the mean-variance portfolio. Gray shaded areas represent NBER recession periods.

Figure 6 shows that the difference between the average traditional and achievable alphas widens during periods of financial turmoil such as the two back-to-back recessions in 1980 and 1981–82, the dot-com bubble of the early 2000's, and the Great Financial Crisis of

Figure 6: Time series of traditional and achievable alpha

This figure depicts the time series of the cross-sectional average traditional and achievable alpha computed on a 36-month rolling window for the seven models in Table 2. The traditional alpha is computed with respect to all long-short factors for each model and the achievable alpha with respect to just those long-only factors that have a positive weight in the mean-variance portfolio. Gray-shaded areas represent NBER recession periods.



2009. This implies that the difference between traditional and achievable alpha is particularly significant during periods of financial crises, when shortsale constraints are likely to be more binding. An implication of this result, is that shortsale-constrained investors should use achievable alpha to evaluate mutual-fund performance *particularly* during financial crises.

We now formally estimate the relationship between (i) the difference between the traditional alpha and the achievable alpha, and (ii) market risk, using the following panel regression:

$$\Delta\alpha_{mf,b,t} = \beta \cdot \text{Risk}_t + B_b + \text{MF}_{mf} + \varepsilon_{mf,b,t}, \tag{9}$$

where $\Delta\alpha_{mf,b,t}$ is the difference between the traditional and achievable alpha of mutual fund mf estimated under model b at time t, Risk_t is the market volatility estimated from monthly market returns over the prior 36-month period at time t, B_b represents model fixed effects to

Table 8: Difference in alphas during periods of financial turmoil

This table reports the slope coefficient for the market-risk variable in (9), along with its standard error and the regression R-squared. The independent variable is standardized, allowing the coefficients to be interpreted as the change in the difference between the traditional and achievable alpha (in basis points) resulting from a one-standard-deviation increase in market volatility. The first seven columns report the results for pooled regressions for each model, and the eighth column across all models. Model fixed effects are included only for the pooled regression across all models. Standard errors are clustered by fund.

	CAPM	FFC	FF5	FF6	HXZ	HXZ	VANG	ALL
Slope (bps)	8.189	5.589	10.011	10.076	11.770	12.875	8.301	9.544
Standard errors (bps)	0.073	0.241	0.245	0.254	0.276	0.266	0.218	0.194
Model FE	No	No	No	No	No	No	No	Yes
Fund FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R-squared (%)	7.217	1.993	5.146	5.049	8.789	10.449	4.787	5.429

account for systematic differences across benchmark models, MF_{mf} represents mutual fund fixed effects to capture fund-specific characteristics, and $\varepsilon_{mf,b,t}$ is the idiosyncratic error term.

Table 8 reports the slope coefficient for the market-risk variable in (9), along with its standard error and the regression R-squared. The independent variable is standardized, allowing the coefficients to be interpreted as the change in the difference between the traditional and achievable alpha (in basis points) resulting from a one-standard-deviation increase in market volatility. The first seven columns report the results for pooled regressions for each model, and the eighth column across all models. Table 8 shows that the effect of a one-standard-deviation increase in market volatility on the difference between the traditional and achievable alphas is significantly positive for every model, ranging between 5.589 basis points for the FFC model and 12.875 basis points for the HXZM model. This confirms the observation from Figure 6 that the difference between the traditional and achievable alpha increases with market volatility, that is, during periods of financial crises.

4.2 Achievable and Traditional Alphas and Fund Flows

We now study whether past performance (measured by either traditional or achievable alpha) explains future mutual-fund flows. This analysis allows us to test whether investors are using traditional or achievable alpha (or both) when making mutual-fund investment decisions.

We define the flows for fund mf in month t as the percentage growth of new assets:

$$\text{Flow}_{mf,t} = \frac{\text{TNA}_{mf,t} - \text{TNA}_{mf,t-1} \times (1 + R_{rf,t} + R_{mf,t})}{\text{TNA}_{mf,t-1}}, \tag{10}$$

where $\text{TNA}_{mf,t}$ are the total net assets under management of mutual fund mf at the end of month t, $R_{rf,t}$ is the risk-free return, and $R_{mf,t}$ is the mutual-fund return in excess of the risk-free rate.

To study the relation between past performance and fund flows, we run the following panel regression for each benchmark model:

$$\text{Flow}_{mf,t} = a \cdot \alpha_{mf,t-1} + \text{T}_t + \text{MF}_{mf} + \epsilon_{mf,t}, \tag{11}$$

where $\alpha_{mf,t-1}$ is the alpha of fund mf estimated using the returns of the 36 prior months, T_t are time fixed effects, MF_{mf} are mutual-fund fixed effects, and $\epsilon_{mf,t}$ is the error term.[15]

We run panel regression (11) first considering the traditional and achievable alpha individually, and then including both jointly as explanatory variables. Panel A of Table 9 reports the results from estimating the panel regression (11) considering the traditional alpha individually. Consistent with the existing literature, for every model we find that the traditional alpha is highly significant in explaining mutual-fund flows, with t-statistics above 10. Moreover, we also find that, consistent with Barber, Huang, and Odean (2016) and Berk and van Binsbergen (2016), the CAPM traditional alpha explains mutual-fund flows at least as well as any of the other models, with an R-squared value of 2.639%, which is higher than those of the other models.

Panel B of Table 9 reports the results from estimating the panel regression (11) considering the *achievable* alpha individually. As for the traditional alpha, we find that for every model, achievable alpha is highly significant in explaining mutual-fund flows, with t-statistics above 10. In addition, we also find that the CAPM *achievable* alpha explains mutual-fund flows at least as well as the achievable alphas of the other models, with an R-squared value of 2.544%, which is higher than those of the other models. Moreover, we find that the R-squared values in Panel C are higher than those in Panels A and B, which again confirms that both the traditional and achievable alpha contribute to explain fund flows.

To study whether achievable alpha contains information about mutual-fund flows that is independent from that contained in the traditional alpha, Panel C of Table 9 reports the results from estimating the panel regression (11) considering *jointly* the traditional and

[15] We account for time and fund fixed effects by first subtracting the time-series average and then the cross-sectional average to our flow and alpha variables.

Table 9: Achievable and Traditional Alphas and Fund Flows

This table reports slope coefficients and their t-statistics for several versions of panel regression (11). Panel A considers the regression of fund flows on traditional alpha, Panel B the regression of fund flows on achievable alpha, Panel C the regression of fund flows jointly on traditional and achievable alphas, and Panel D the regression of fund flows jointly on traditional and achievable alphas and their interactions with a "Risk" indicator variable that takes a value of one for months in the top decile of months sorted by realized market volatility in the prior 36 months, and zero otherwise. For all regressions, we consider time and fund fixed effects, control for lagged values of fund flows up to 12 months, and double-cluster standard errors by time and fund.

	CAPM	FFC	FF5	FF6	HXZ	HXZM	VANG
Panel A: Traditional alpha							
Slope	0.162	0.157	0.141	0.134	0.149	0.150	0.137
	[29.561]	[37.011]	[28.701]	[28.825]	[22.838]	[32.957]	[27.877]
R2 (%)	2.639	2.476	1.987	1.802	2.214	2.253	1.874
Panel B: Achievable alpha							
Slope	0.159	0.148	0.153	0.149	0.155	0.152	0.146
	[21.728]	[21.203]	[21.559]	[21.435]	[21.585]	[21.628]	[19.980]
R2 (%)	2.544	2.182	2.350	2.233	2.408	2.320	2.117
Panel C: Traditional and achievable alpha							
Slope $\alpha_{\mathcal{T}}$	0.137	0.106	0.073	0.071	0.079	0.088	0.064
	[4.832]	[14.447]	[10.607]	[10.476]	[9.168]	[12.674]	[7.162]
Slope $\alpha_{\mathcal{A}}$	0.026	0.073	0.107	0.108	0.100	0.094	0.098
	[0.946]	[9.608]	[14.471]	[15.035]	[12.446]	[12.781]	[9.703]
R2 (%)	2.642	2.738	2.673	2.558	2.737	2.754	2.295
Panel D: Traditional and achievable alpha with risk-interaction terms							
Slope $\alpha_{\mathcal{T}}$	0.147	0.109	0.077	0.074	0.084	0.094	0.070
	[4.929]	[14.782]	[10.803]	[10.624]	[9.504]	[13.704]	[7.590]
Slope $\alpha_{\mathcal{A}}$	0.014	0.070	0.103	0.104	0.095	0.089	0.092
	[0.501]	[9.081]	[13.794]	[14.367]	[11.847]	[12.092]	[8.863]
Slope $\alpha_{\mathcal{T}}\times$Risk	-0.144	-0.020	-0.027	-0.023	-0.025	-0.034	-0.026
	[-3.695]	[-2.039]	[-3.733]	[-3.240]	[-3.103]	[-4.812]	[-2.858]
Slope $\alpha_{\mathcal{A}}\times$Risk	0.148	0.019	0.027	0.024	0.030	0.034	0.025
	[3.853]	[2.129]	[3.570]	[3.366]	[3.773]	[4.803]	[2.658]
R2 (%)	2.671	2.749	2.708	2.586	2.780	2.813	2.318

achievable alphas. Our main finding is that both the traditional and achievable alphas are generally highly significant, with t-statistics exceeding five across models, except for the CAPM. This finding demonstrates that the traditional and achievable alphas contain independent information and suggests that at least some investors use achievable alpha to make investment decisions.

To examine whether the relative importance of traditional and achievable alphas to explain fund flows depends on market conditions, Panel D of Table 9 reports the results from estimating a panel regression that considers jointly the traditional and achievable alphas and includes *also* their interactions with a "Risk" indicator variable that takes a value of one for months in the top decile of months sorted by realized market volatility in the prior 36 months, and zero otherwise. We find that the sensitivity of fund flows to traditional alpha generally *weakens* during periods of elevated market volatility, with the coefficient for the interaction between the past traditional alpha and the Risk indicator variable being negative for every model and significant for every model except FFC. In contrast, consistently across all models, the relation between fund flows and achievable alpha *strengthens* when market volatility is high, with the coefficient for the interaction between the past achievable alpha and the Risk indicator variable being significantly positive for every model except FFC. These findings suggest that investors find that achievable alpha is a more informative measure of future performance during periods of financial turmoil, and thus, they allocate more capital to funds with high achievable alpha during periods of high market volatility.

5 Relaxing the Shortsale Constraints

In the previous sections, we evaluated the achievable alpha for an investor who cannot short the benchmark factors. While such shortsale constraints represent a realistic scenario for most retail and institutional investors, such as certain pension funds, we now evaluate the performance of mutual funds for investors that can engage in a *limited* amount of shortselling. Section 5.1 considers the case of an investor who can short *only* the market factor using, for instance, an inverse market ETF. Section 5.2 considers the case of an investor who can short the benchmark factors but faces a leverage constraint.

5.1 Shorting the Market

Since 2006, investors can short the market by buying an inverse ETF. Although inverse ETFs represent a minuscule fraction of the ETF industry,[16] it is informative to evaluate the performance of mutual funds for an investor who can short the market.

[16] As of November 2024, the ETF database lists around 72 inverse ETFs with an aggregate AUM of only around $8.4 billion; see https://etfdb.com/etfdb-category/inverse-equities/.

Table 10: Fund performance for investors who can short the market

This table reports cross-sectional statistics for the achievable fund alpha and value-added with respect to the long-only version of the seven factor models listed in Table 2 for investors who can short the market. For each fund, Panels A and B report the achievable alpha and value-added with respect to the market plus other benchmark factors with positive weight in the mean-variance portfolio of an investor who can short only the market. We report the average alpha and value-added across funds, their t-statistics, the time-weighted average achievable alpha and value-added, where the weight is proportional to the length of the sample period for which we have return data for the fund, and their t-statistics. We also report the percentiles of the cross-sectional distributions of achievable fund alpha and value-added, the percentage of funds with positive achievable alpha and value-added, and the percentage of funds with achievable alpha and value-added t-statistic greater than two. Alphas are annualized and reported in percentage. Like Barras et al. (2022), we winsorize observations that are more than five times the inter-decile range (difference between the 90th and 10th percentiles) away from the median.

	CAPM	FFC	FF5	FF6	HXZ	HXZM	VANG
Panel A: Achievable alpha with respect to long-only factors							
Average alpha	0.61	-0.29	-0.39	-0.63	-1.20	-1.20	-0.58
t-stat	11.68	-5.14	-7.54	-11.00	-16.70	-16.72	-9.02
Time-weighted average alpha	0.87	0.06	-0.08	-0.29	-0.81	-0.81	0.07
t-stat	4.28	1.26	-1.71	-3.55	-4.16	-4.16	1.26
10th percentile	-2.23	-3.31	-3.25	-3.93	-5.29	-5.28	-4.00
50th percentile	0.59	0.12	-0.18	-0.22	-0.56	-0.56	-0.23
90th percentile	3.33	2.43	2.22	2.11	2.19	2.19	2.56
Percentage of funds with $\alpha > 0$	62.76	52.18	46.59	44.94	40.41	40.28	46.19
Percentage of funds with $t(\alpha) > 2$	7.89	8.19	4.14	5.69	5.82	5.95	6.16
Panel B: Achievable value-added with respect to long-only factors							
Average value-added	-1.71	-2.66	-3.83	-3.86	-7.65	-7.63	-5.80
t-stat	-4.85	-7.56	-10.31	-10.52	-15.31	-15.30	-14.04
Time-weighted average value-added	-0.87	-2.17	-3.82	-3.83	-8.22	-8.21	-4.73
t-stat	-1.90	-3.43	-3.97	-3.99	-4.23	-4.23	-4.07
10th percentile	-13.92	-16.17	-17.25	-18.20	-27.73	-27.55	-23.09
50th percentile	-0.81	-1.14	-1.67	-1.63	-2.78	-2.76	-2.20
90th percentile	8.15	7.22	5.60	6.22	4.62	4.62	5.35
% funds with value-added>0	38.37	35.62	29.92	30.73	26.99	26.99	26.30

It is straightforward to extend Proposition 2 to show that the marginal utility improvement for an investor (who can short only the market) when she has access to a mutual fund in addition to the benchmark factors is measured by the achievable alpha, defined as the fund's alpha with respect to the return of the market factor plus the returns of other benchmark factors with positive weight in the investor's mean-variance portfolio. Similarly, the achievable value-added can be computed using the mutual fund's abnormal returns with respect to the market and other benchmark factors with positive weight in the investor's portfolio. For the VANG model, we allow the investor to assign a positive or negative weight to VFINX, which is the Vanguard fund tracking the S&P500 index.

Panels A and B of Table 10 report the achievable alpha and value-added of an investor with access to long-only benchmark factors and can short only the market. Consistent with the results in Section 3.4, we find that the achievable alpha and value-added of this investor are much smaller than the traditional alpha and value-added of an unconstrained investor who has access to long-short benchmark factors, which we report in Panel A of Tables 5 and 6. However, we also note that the percentage of funds with positive achievable alpha and value-added increases substantially when the investor can short the market. For instance, when the investor cannot short the market, the percentage of mutual funds with positive achievable alpha with respect to the long-only version of the factor models ranges between 11.72% for HXZM and 37.27% for VANG. In contrast, when the investor can short the market, we have that the percentage of mutual funds with positive achievable alpha with respect to the long-only version of the models ranges between 40.28% for HXZM and 52.18% for FFC. Similarly, when the investor cannot short the market, the percentage of mutual funds with positive achievable value-added with respect to the long-only models ranges between 8.90% for HXZM and 20.13% for FFC. In contrast, when the investor can short the market, the percentage of mutual funds with positive achievable value-added with respect to the long-only models ranges between 26.99% for HXZM and 35.62% for FFC. An implication of the results in Table 10 is that even though inverse ETFs represent only a small fraction of the ETF market, they have the potential to substantially improve the mean-variance efficiency gains offered by active mutual funds to shortsale-constrained investors.

Table 11 reports the difference between the rankings of mutual funds based on the traditional and achievable alpha and value-added with respect to the long-only version of the factor models for investors who can short only the market. The table shows that while the ability of investors to short the market can help them to benefit from investing in active mutual funds, the relative ranking of mutual funds for such investors continues to be very sensitive to the presence of shortsale constraints on the other benchmark factors. To see this, note that Table 11 shows that the ranking of more than 60% of the funds changes by at least one decile for every factor model except CAPM.

Table 11: Difference in fund rankings for investors who can short the market

This table reports the difference ("Diff") between the rankings of mutual funds based on the traditional and achievable alpha and value-added with respect to the long-only version of the seven factor models listed in Table 2 for investors who can short only the market. Panel A reports the ranking difference in terms of traditional and achievable alphas, and Panel B in terms of traditional and achievable value-added. For each factor model, we report the measure Diff defined in Equation (7) in percentage.

	CAPM	FFC	FF5	FF6	HXZ	HXZM	VANG
Panel A: Difference in rankings based on alpha							
Diff (%)	0.00	68.28	75.00	78.58	82.93	83.75	74.01
Panel B: Difference in rankings based on value-added							
Diff (%)	0.00	61.03	71.39	72.31	75.53	76.42	78.45

5.2 Leverage-Constrained Investors

We now consider the case of a mean-variance investor who can short the benchmark factors, but faces a leverage constraint. Specifically, we consider an investor whose aggregate short position has to be smaller than a fraction δ of her aggregate long position. The portfolio selection problem of this investor can be formulated as:

$$\max_{w_b, w_{mf}} \quad \text{MVU}(w_b, w_{mf}) \tag{12}$$

$$\text{s.t.} \quad w_b + \psi_s - \psi_\ell = 0, \tag{13}$$

$$w_{mf} + \nu_s - \nu_\ell = 0, \tag{14}$$

$$e^\top \psi_s + \nu_s \leq \delta(e^\top \psi_\ell + \nu_\ell), \tag{15}$$

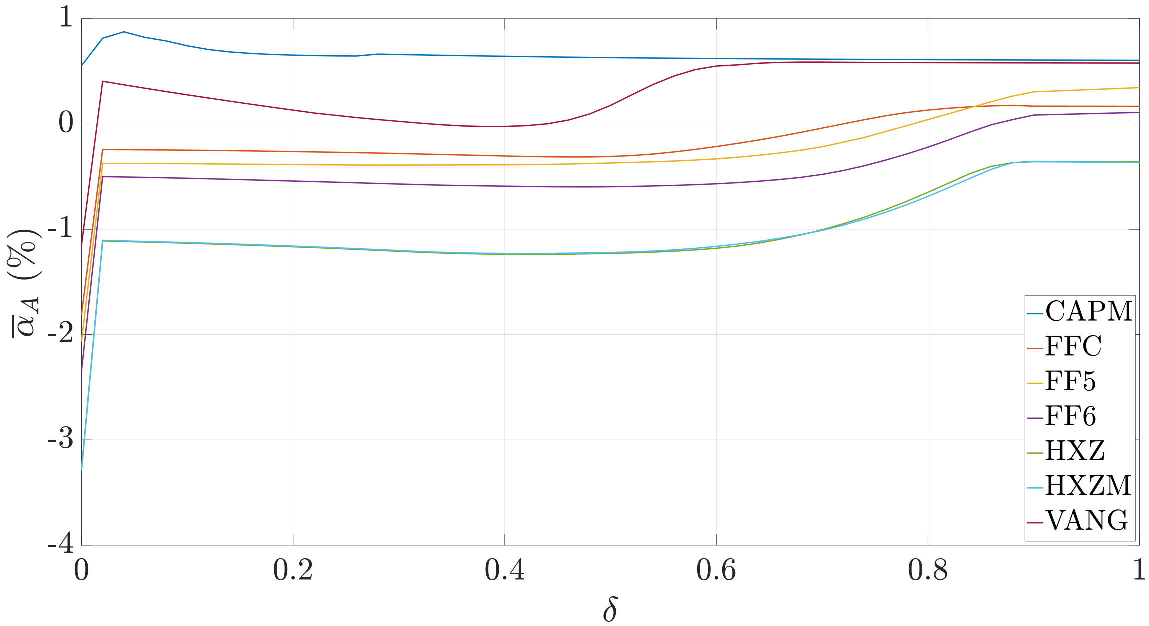
$$\psi_\ell,\ \psi_s,\ \nu_\ell,\ \nu_s \geq 0, \tag{16}$$

where w_b is the vector of benchmark factor weights, w_{mf} is the weight on the mutual fund, ψ_s and ψ_ℓ are slack variables that measure the negative and positive positions of the benchmark portfolio, ν_s and ν_ℓ are slack variables that measure the negative and positive positions on the mutual fund, e is the vector of ones, the constraint $e^\top \psi_s + \nu_s \leq \delta(e^\top \psi_\ell + \nu_\ell)$ requires that the aggregate short position of the investor is smaller than a fraction δ of her aggregate long position, and the investor's mean-variance utility is

$$\text{MVU}(w_b, w_{mf}) = \begin{bmatrix} \mu_b^\top & \mu_{mf}^\top \end{bmatrix} \begin{bmatrix} w_b \\ w_{mf} \end{bmatrix} - \frac{\gamma}{2} \begin{bmatrix} w_b^\top & w_{mf}^\top \end{bmatrix} \begin{bmatrix} \Sigma_b & \Sigma_{b,mf} \\ \Sigma_{mf,b} & \Sigma_{mf} \end{bmatrix} \begin{bmatrix} w_b \\ w_{mf} \end{bmatrix}. \tag{17}$$

Figure 7: Achievable alpha for leverage-constrained investors

This figure depicts the cross-sectional average achievable alpha for leverage-constrained investors, computed using Proposition 4, for different values of the leverage parameter δ.



For a given w_{mf}, the optimal portfolio of the benchmark factors for the leverage-constrained investor is given by the following portfolio selection problem:

$$\text{MVU}^*(w_{mf}) = \max_{w_b} \quad \text{MVU}(w_b, w_{mf}) \tag{18}$$

$$\text{s.t.} \quad w + \psi_s - \psi_\ell = 0, \tag{19}$$

$$w_{mf} + \nu_s - \nu_\ell = 0, \tag{20}$$

$$e^\top \psi_s + \nu_s \leq \delta(e^\top \psi_\ell + \nu_\ell), \tag{21}$$

$$\psi_\ell,\ \psi_s,\ \nu_\ell,\ \nu_s \geq 0, \tag{22}$$

where $\text{MVU}^*(w_{mf})$ is the optimal mean-variance utility as a function of the weight on the mutual fund w_{mf}.

In the following proposition, we characterize the marginal mean-variance utility improvement that a leverage-constrained investor can achieve by investing in the fund in addition to the benchmark factors.

Proposition 4 *Let the leverage-constrained mean-variance portfolio of the benchmark factors be w_b^*, that is, let w_b^* be the solution to Problem (18) for $w_{mf} = 0$. Then, the marginal* *mean-variance utility improvement that a leverage-constrained investor can achieve by investing in the fund in addition to the benchmark factors is the achievable alpha,*

$$\alpha_{\mathcal{A}} = \mu_{mf} - \gamma \Sigma_{mf,b} w_b^* - \lambda, \tag{23}$$

where μ_{mf} is the mean gross return of the fund, γ is the investor's relative risk aversion, $\Sigma_{mf,b}$ is the covariance between the mutual fund return and the benchmark factor returns, and λ is the Lagrange multiplier for the constraint $w_{mf} + \nu_s - \nu_\ell = 0$ for problem (18) at the maximizer w_b^* for the value of $w_{mf} = 0$.

Figure 7 depicts, using Proposition 4, for different values of the leverage parameter δ, the cross-sectional average achievable alpha for leverage-constrained investors. Figure 7 shows that the average achievable alpha of a leverage-constrained investor is substantially smaller than the traditional alpha of an unconstrained investor. For instance, for the case with relatively high leverage of $\delta = 0.4$, the average achievable alpha remains negative for all factor models except CAPM. For the case with $\delta = 1$, for which the investor can hold aggregate negative positions as large as her aggregate positive position, the average achievable alpha remains negative for the HXZ and HXZM models but is positive for the other models.

6 Conclusion

The traditional approach to evaluating mutual-fund performance is to compute the fund's alpha with respect to a set of benchmark factors. However, if the benchmark-factor portfolio includes short positions in some factors, then this alpha is unachievable for shortsale-constrained investors. We propose a simple approach to evaluate mutual-fund performance for such investors.

Theoretically, we show that the marginal-utility gain that a shortsale-constrained investor can achieve when she has access to a mutual fund in addition to the benchmark factors can be measured by the *achievable alpha*, which is the fund alpha with respect to only those benchmark factors that have a strictly positive weight in the shortsale-constrained mean-variance portfolio. Empirically, we find that mutual-fund performance substantially deteriorates when assessed using achievable alpha or value added: while 62.54% and 44.58%

of funds have positive traditional alpha and value-added, only 37.27% and 18.38% have positive achievable alpha and value-added for a benchmark containing eight Vanguard funds.

Intuitively, dropping some factors should worsen the performance of the benchmark portfolio, and thus, one would expect the alpha of a mutual fund to be larger with respect to the restricted benchmark. However, we show theoretically that if a fund has positive exposure to some of the factors with a zero weight in the shortsale-constrained mean-variance portfolio, then the achievable alpha can be smaller than the traditional alpha. Empirically we find that this indeed is the case. We also relate the gap between the traditional and achievable alphas to macroeconomic conditions and find it widens during periods of financial turmoil.

An important implication of our work is that the value of active fund management for shortsale-constrained investors is substantially smaller than previously thought. Thus, retail investment platforms and pension funds that (by mandate or strategy) abstain from shorting assets should evaluate mutual-fund performance in terms of achievable alpha or value-added to account for their client's constraints.

A Appendix: Proofs of Propositions

In this section, we provide the proof for each proposition in the main text.

A.1 Proof of Proposition 1

Let us define the return of a portfolio that combines the return from the mean-variance portfolio combination of the benchmark factors and the return of the fund. This is,

$$R_{p,t} = w_b^\top R_{b,t} + w_{mf} R_{mf,t}, \tag{A1}$$

where $R_{b,t}$ is the K-dimensional vector of benchmark excess returns at time t with mean μ_b and covariance matrix Σ_b, w_b is the portfolio of benchmark factors, $R_{mf,t}$ is the before-fees mutual-fund excess return at time t, and w_{mf} is the weight on the fund. In addition, and without loss of generality, the fund return is defined according to a linear factor model as

$$R_{mf,t} = \alpha_{\mathcal{T}} + \beta R_{b,t} + \epsilon_{b,t}, \tag{A2}$$

where $\epsilon_{b,t}$ is a zero-mean random variable with standard deviation σ_ϵ. Therefore, we can redefine the portfolio return as

$$R_{p,t} = (\underbrace{w_b + w_{mf}\beta_i}_{=\widetilde{w}_b})^\top R_{b,t} + w_{mf}(\alpha_{\mathcal{T}} + \epsilon_{b,t}). \tag{A3}$$

Because $R_{b,t}$ and $(\alpha_{\mathcal{T}} + \epsilon_{b,t})$ are uncorrelated, we can optimize $\widetilde{w}_b$ and w_{mf} independently. Accordingly, define the investor's mean-variance utility as

$$\mathrm{E}\left[\widetilde{w}_b^\top R_{b,t} + w_{mf}(\alpha_{\mathcal{T}} + \epsilon_{b,t})\right] - \frac{\gamma}{2}\mathrm{Var}\left[\widetilde{w}_b^\top R_{b,t} + w_{mf}(\alpha_{\mathcal{T}} + \epsilon_{b,t})\right], \tag{A4}$$

where γ is the investor's risk aversion parameter. We now provide the derivative of the investor's mean-variance utility with respect to w_{mf}:

$$\frac{\partial \mathrm{E}\left[\widetilde{w}_b^\top R_{b,t} + w_{mf}(\alpha_{\mathcal{T}} + \epsilon_{b,t})\right] - \frac{\gamma}{2}\mathrm{Var}\left[\widetilde{w}_b^\top R_{b,t} + w_{mf}(\alpha_{\mathcal{T}} + \epsilon_{b,t})\right]}{\partial w_{mf}} = \alpha_{\mathcal{T}} - \gamma w_{mf}\sigma_\epsilon^2, \tag{A5}$$

and evaluating the derivative at $w_{mf} = 0$ gives $\alpha_{\mathcal{T}}$, which completes the proof. □

A.2 Proof of Proposition 2

The investor's mean-variance utility for any portfolio w is

$$\text{MVU}(w) = \begin{bmatrix} \mu_{b_+}^\top & \mu_{b_0}^\top & \mu_{mf}^\top \end{bmatrix} w - \frac{\gamma}{2} w^\top \begin{bmatrix} \Sigma_{b_+} & \Sigma_{b_+,b_0} & \Sigma_{b_+,mf} \\ \Sigma_{b_0,b_+} & \Sigma_{b_0} & \Sigma_{b_0,mf} \\ \Sigma_{mf,b_+} & \Sigma_{mf,b_0} & \Sigma_{mf} \end{bmatrix} w.$$

Once the investor has access to the fund, the portfolio $w_0 = (w_{b_+}^*, w_{b_0}^* = 0, w_{mf} = 0)$ is no longer mean-variance efficient for her. Assuming the investor is currently holding the shortsale-constrained mean-variance portfolio of the benchmark factors w_0, she would maximize the marginal improvement to her mean-variance utility by shifting her portfolio in the direction of the gradient of her mean-variance utility evaluated at w_0, that is, by shifting her portfolio as follows:

$$w = w_0 + \delta \nabla_w \text{MVU}(w_0), \tag{A6}$$

where δ is infinitesimally small and $\nabla_w \text{MVU}(w_0)$ is the gradient of the investor's mean-variance utility evaluated at w_0. Moreover,

$$\nabla_w \text{MVU}(w) = \begin{bmatrix} \mu_{b_+} \\ \mu_{b_0} \\ \mu_{mf} \end{bmatrix} - \gamma \begin{bmatrix} \Sigma_{b_+} & \Sigma_{b_+,b_0} & \Sigma_{b_+,mf} \\ \Sigma_{b_0,b_+} & \Sigma_{b_0} & \Sigma_{b_0,mf} \\ \Sigma_{mf,b_+} & \Sigma_{mf,b_0} & \Sigma_{mf} \end{bmatrix} \begin{bmatrix} w_{b_+} \\ w_{b_0} \\ w_{mf} \end{bmatrix}. \tag{A7}$$

Therefore, the gradient evaluated at w_0 is

$$\nabla_w \text{MVU}(w_0) = \begin{bmatrix} \mu_{b_+} \\ \mu_{b_0} \\ \mu_{mf} \end{bmatrix} - \gamma \begin{bmatrix} \Sigma_{b_+} & \Sigma_{b_+,b_0} & \Sigma_{b_+,mf} \\ \Sigma_{b_0,b_+} & \Sigma_{b_0} & \Sigma_{b_0,mf} \\ \Sigma_{mf,b_+} & \Sigma_{mf,b_0} & \Sigma_{mf} \end{bmatrix} \begin{bmatrix} w_{b_+}^* \\ 0 \\ 0 \end{bmatrix} \tag{A8}$$

$$= \begin{bmatrix} \mu_{b_+} - \gamma \Sigma_{b_+} w_{b_+}^* \\ \mu_{b_0} - \gamma \Sigma_{b_0,b_+} w_{b_+}^* \\ \mu_{mf} - \gamma \Sigma_{mf,b_+} w_{b_+}^* \end{bmatrix} \tag{A9}$$

Note that $w_b^* = (w_{b_+}^*, w_{b_0}^* = 0)$ is the shortsale-constrained mean-variance portfolio for the case where the investor does not have access to the fund. Thus, we must have that $\mu_{b_+} - \gamma \Sigma_{b_+} w_{b_+}^* = 0$, $\mu_{b_0} - \gamma \Sigma_{b_0,b_+} w_{b_+}^* \leq 0$, and $w_{b_+}^* = \frac{1}{\gamma} \Sigma_{b_+}^{-1} \mu_{b_+}$. Consequently

$$\nabla_w \text{MVU}(w_0) = \begin{bmatrix} 0 \\ \mu_{b_0} - \gamma \Sigma_{b_0,b_+} w_{b_+}^* \leq 0 \\ \mu_{mf} - \Sigma_{mf,b_+} \Sigma_{b_+}^{-1} \mu_{b_+} \end{bmatrix}. \tag{A10}$$

Furthermore, note that $\alpha_{\mathcal{A}} = \mu_{mf} - \Sigma_{mf,b_+}\Sigma_{b_+}^{-1}\mu_{b_+}$ is the alpha of the fund with respect to the b_+ benchmark factors, which have a strictly positive weight in the shortsale-constrained mean-variance portfolio. Therefore, for an investor with relative risk aversion γ, we have that

$$\nabla_w \text{MVU}(w_0) = \begin{bmatrix} 0 \\ \mu_{b_0} - \gamma\Sigma_{b_0,b_+}w^*_{b_+} \leq 0 \\ \alpha_{\mathcal{A}} \end{bmatrix}. \tag{A11}$$

Note that the gradient of the investor's mean-variance utility with respect to w_{b_+} is zero, which means she has no incentive to change her weight on the b_+ factors, the gradient of her mean-variance utility with respect to the b_0 factors is negative, which implies (because of shortsale constraints) that she cannot reduce the weight on the b_0 factors, and the gradient of her mean-variance utility with respect to the weight on the fund is equal to the fund alpha $\alpha_{\mathcal{A}}$. This shows that, to maximize the marginal improvement to her mean-variance utility, the investor should increase her weight on the fund while keeping the weights on the benchmark factors fixed at $w^*_b = (w^*_{b_+}, w^*_{b_0} = 0)$, and the marginal improvement to her mean-variance utility per dollar invested in the fund would be $\alpha_{\mathcal{A}}$. □

A.3 Proof of Proposition 3

First, without loss of generality, we define the factors that the shortsale-constrained mean-variance portfolio assigns a weight of zero as in Equation (4). Second, we define the shortsale-constrained mean-variance portfolio of all the K benchmark factors as

$$w_b = \Sigma_b^{-1}[\mu_b + \eta_b], \tag{A12}$$

where $\eta_b \geq 0 \in R^K$ is the vector of Lagrange multipliers associated with the non-negativity constraints. The partition covariance matrix Σ_b is

$$\Sigma_b = \begin{bmatrix} \Sigma_{b_+} & \Sigma_{b_+,b_0} \\ \Sigma_{b_+,b_0}^\top & \Sigma_{b_0} \end{bmatrix}, \tag{A13}$$

where $\Sigma_{b_+} \in R^{K_+ \times K_+}$ is the covariance matrix for the factors $R_{b_+,t}$ for which the shortsale-constrained mean-variance portfolio assigns a positive weight, $\Sigma_{b_0} \in R^{K_0 \times K_0}$ is the covariance matrix for the factors $R_{b_0,t}$ for which the shortsale-constrained mean-variance portfolio assigns a zero weight, and $\Sigma_{b_+,b_0} \in R^{K_+ \times K_0}$ is the covariance matrix between the $R_{b_+,t}$ and

$R_{b_0,t}$ factors. The partitioned vector of means is

$$\mu_b = \begin{bmatrix} \mu_{b_+} \\ \mu_{b_0} \end{bmatrix}, \tag{A14}$$

where $\mu_{b_+} \in R^{K_+}$ is the vector of mean returns for the factors $R_{b_+,t}$ for which the shortsale-constrained mean-variance portfolio assigns a positive weight and $\mu_{b_0} \in R^{K_0}$ is the vector of mean returns for the factors $R_{b_0,t}$ for which the shortsale-constrained mean-variance portfolio assigns a zero weight. Using the definition for the partitioned inverse covariance matrix

$$\Sigma_b^{-1} = \begin{bmatrix} \Sigma_{b_+}^{-1} + \Sigma_{b_+}^{-1}\Sigma_{b_+,b_0}S^{-1}\Sigma_{b_+,b_0}^{\top}\Sigma_{b_+}^{-1} & -\Sigma_{b_+}^{-1}\Sigma_{b_+,b_0}S^{-1} \\ -S^{-1}\Sigma_{b_+,b_0}^{\top}\Sigma_{b_+}^{-1} & S^{-1} \end{bmatrix}, \tag{A15}$$

where $S = \Sigma_{b_0} - \Sigma_{b_+,b_0}^{\top}\Sigma_{b_+}^{-1}\Sigma_{b_+,b_0}$, we can obtain the closed-form expression for the weights assigned to the factors $R_{b_0,t}$. This is

$$w_{b_0} = S^{-1}\big[\underbrace{\mu_0 - \Sigma_{b_+,b_0}^{\top}\Sigma_{b_+}^{-1}\mu_+}_{\alpha_{0,+}} \underbrace{-\Sigma_{b_+,b_0}^{\top}\Sigma_{b_+}^{-1}\eta_+ + \eta_0}_{\eta_0}\big]. \tag{A16}$$

The first underbrace bracket comes from the fact that $\Sigma_{b_+,b_0}^{\top}\Sigma_{b_+}^{-1} = \phi$ in Equation (4), and the second underbrace bracket comes from the fact that $\eta_+ = 0$, which is the vector of Lagrange multipliers associated to the factors with a positive weight. Thus, we have that

$$w_{b_0} = S^{-1}[\alpha_{0,+} + \eta_0] = 0, \tag{A17}$$

because w_{b_0} is the vector of weights for which the shortsale-constrained mean-variance portfolio finds optimal to assign a zero weight. Pre-multiplying w_{b_0} with S, we have that

$$[\alpha_{0,+} + \eta_0] = 0, \tag{A18}$$

which implies that $\alpha_{0,+} = -\eta_0$. Because $\eta_0 > 0$ is the vector of Lagrange multipliers associated with the non-negativity constraints of the factors for which the shortsale-constrained mean-variance portfolio assigns a zero weight, we have that the vector of intercepts in Equation (4) are all negative.

In the second part of the proof, we show the mechanism behind the alpha decay experienced when we replace the benchmark model without shortsale constraints with a more parsimonious factor model that drops the factor for which the shortsale-constrained mean-variance portfolio assigns a zero weight. To do that, we plug in the equation for the traditional alpha, the expression for the factors $R_{b_0,t}$ in Equation (4). This yields:

$$\alpha_{\mathcal{T}} = R_{mf,t} - \beta_{\mathcal{T},+} R_{b_+,t} - \beta_{\mathcal{T},0} \left(\alpha_{0,+} + \beta_{0,+} R_{b_+,t} + \epsilon_{0,+,t} \right) - \epsilon_{b,t}. \tag{A19}$$

Rearranging terms, we have

$$\alpha_{\mathcal{T}} = \underbrace{R_{mf,t} - \overbrace{(\beta_{\mathcal{T},+} + \beta_{\mathcal{T},0}\beta_{0,+})}^{\tilde{\beta}_{\mathcal{A}}} R_{b_+,t}}_{\alpha_{\mathcal{A}} + \tilde{\epsilon}_{b,t}} - \beta_{\mathcal{T},0}\alpha_{0,+} - \beta_{\mathcal{T},0}\epsilon_{0,+,t} - \epsilon_{b,t}, \tag{A20}$$

Taking expectations, we obtain

$$\alpha_{\mathcal{T}} - \alpha_{\mathcal{A}} = -\beta_{\mathcal{T},0}\alpha_{0,+}. \tag{A21}$$

□

A.4 Proof of Proposition 4

By the envelope theorem (Mas-Colell, Whinston, and Green, 1995, p. 965) we have that

$$\frac{d\text{MVU}^*(w_{mf})}{dw_{mf}} = \left. \frac{\partial \text{MVU}(w_b, w_{mf})}{\partial w_{mf}} \right|_{w_b(w_{mf})} - \lambda,$$

where $w_b(w_{mf})$ is the optimal leverage-constrained mean-variance portfolio of the benchmark factors for a given weight on the mutual fund factor w_{mf}, and λ is the Lagrange multiplier for the constraint $w_{mf} + \nu_s - \nu_\ell = 0$ for the problem in (18) at the maximizer $w_b(w_{mf})$. Moreover,

$$\left. \frac{d\text{MVU}^*(w_{mf})}{dw_{mf}} \right|_{w_{mf}=0} = \left. \frac{\partial \text{MVU}(w_b, w_{mf})}{\partial w_{mf}} \right|_{w_b^*, w_{mf}=0} - \lambda = \mu_{mf} - \gamma \Sigma_{mf,b} w_b^* - \lambda,$$

where the second equality follows by taking the partial derivative of the right-hand side of Equation (17) with respect to w_{mf}. □

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Internet Appendix to

Rethinking Mutual Fund Performance: From Traditional Alpha to Achievable Alpha

This Internet Appendix reports the following robustness checks and additional results: (i) evaluating achievable skill and scale and (ii) studying the explanation for the decay of achievable alpha.

IA.1 Scale and Scalability

In the main body of the manuscript, we study two traditional performance measures—alpha and value-added—that are commonly used to evaluate fund performance. However, it is also informative to analyze the underlying skill of a fund and the scalability of its investment strategy. Barras et al. (2022) define a fund's alpha as a function of its skill and scalability. Specifically, they define a fund's gross alpha as

$$\alpha_{mf} = a_{mf} - b_{mf} \times \overline{\text{TNA}}_{mf}, \qquad \text{(IA1)}$$

where a_{mf} and b_{mf} are the skill and scale parameters for fund mf, and $\overline{\text{TNA}}_{mf}$ is the average total net assets (in year 2000 dollars) of fund mf. While a larger a_{mf} indicates higher skill, a larger b_{mf} implies lower scalability.

We now examine the skill and scalability parameters a_{mf} and b_{mf} for both the traditional and the achievable alphas. Table IA.1 reports cross-sectional statistics for these parameters. Panel A shows that the average skill decreases when computed using the traditional alpha. We also observe that although the percentage of funds with positive skill remains above 50% in all models, it is lower than the percentage derived from the traditional alpha. Panel B shows that the scale parameter b increases substantially when estimated from the achievable alpha. This increase in b suggests that the trading strategies of active mutual funds are less scalable, and that their returns decline more steeply as their scale grows. Overall, these findings indicate that accounting for shortsale constraints leads to a deterioration in both skill and scalability.

IA.2 Time-varying risk

In the main body of the manuscript, we follow Berk and van Binsbergen (2015) and compute fund alphas using each fund's entire return history. However, funds are expected to adjust their risk exposure over time. To account for this time variation, we estimate fund alphas using 36-month rolling windows. To compute a fund's alpha, we require a minimum of 30

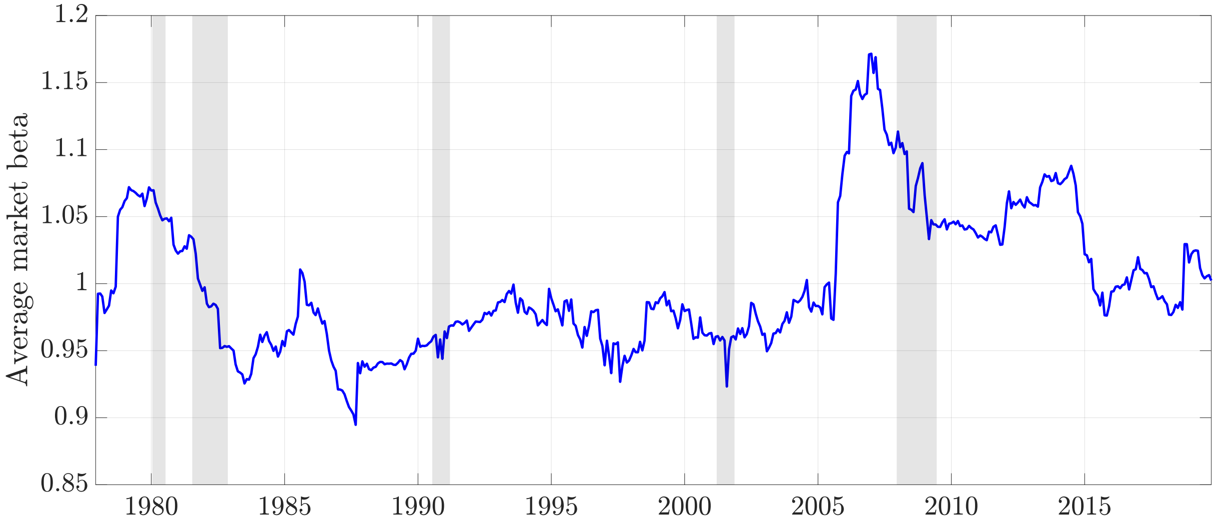
Table IA.1: Achievable skill and scalability

This table reports cross-sectional statistics for the traditional and achievable scale and scalability parameters that define fund alpha in (IA1), with respect to the seven factor models listed in Table 2. For each fund, we compute the traditional alpha by regressing the fund returns on all factors for each model and the achievable alpha on only those factors with a strictly positive weight in the shortsale-constrained mean-variance portfolio over the sample period for which we have return data for the fund. We report the average scale and scalability parameters across funds, their t-statistic (the time-weighted average scale and scalability parameters), where the weight is proportional to the length of the sample period for which we have return data for the fund, and their t-statistic. We also report percentiles of the cross-sectional distribution of fund scale and scalability parameters and the percentage of funds with positive scale and scalability parameters and t-statistic greater than two. Scale and scalability parameters are annualized and reported in percentage. Like Barras et al. (2022), we winsorize observations that are more than five times the inter-decile range (difference between the 90th and 10th percentiles) away from the median.

	CAPM	FFC	FF5	FF6	HXZ	HXZM	VANG
			Panel A: Skill				
Panel A.1: Skill from traditional alpha							
Average	4.37	2.60	2.73	2.56	2.91	2.74	2.79
T-stat	27.91	22.69	21.81	21.47	21.94	21.68	21.67
10th percentile	-2.39	-2.62	-2.69	-2.77	-3.01	-2.93	-3.20
50th percentile	3.29	2.19	2.02	1.95	2.17	2.10	2.40
90th percentile	11.96	8.58	9.37	8.89	9.90	9.28	9.23
Percentage funds a >0	80.80	75.17	70.47	70.52	71.24	70.69	75.42
Panel A.2: Skill from achievable alpha							
Average	4.33	2.38	2.42	1.51	1.27	1.23	2.92
T-stat	27.12	15.83	14.36	10.14	7.96	7.83	16.59
10th percentile	-2.44	-4.71	-4.77	-5.41	-5.80	-5.82	-4.47
50th percentile	3.28	1.97	1.54	1.14	0.56	0.56	2.17
90th percentile	11.90	9.66	10.18	8.44	8.43	8.36	11.10
Percentage funds a >0	80.66	67.76	64.53	61.06	56.83	56.92	67.60
			Panel B: Scale				
Panel B.1: Scale from traditional alpha							
Average	28.84	19.05	18.85	17.83	20.92	19.27	16.72
T-stat	15.75	13.86	13.21	13.44	14.54	13.91	12.01
10th percentile	-8.25	-10.53	-12.84	-12.96	-10.36	-11.64	-13.46
50th percentile	6.43	4.03	3.41	3.31	4.25	3.96	3.95
90th percentile	102.05	72.14	75.17	70.24	77.57	72.93	68.76
Percentage funds b >0	80.90	75.48	72.87	73.15	75.39	74.83	75.57
Panel B.2: Scale from achievable alpha							
Average	29.40	36.21	36.27	30.57	37.74	37.52	34.12
T-stat	15.75	16.21	16.02	14.68	17.28	16.97	14.61
10th percentile	-8.81	-10.86	-11.62	-13.26	-7.20	-7.25	-11.46
50th percentile	6.38	7.34	6.49	5.97	8.00	7.86	6.02
90th percentile	103.09	126.69	122.98	111.26	128.08	127.70	122.20
Percentage funds b >0	80.71	76.69	77.84	76.05	78.90	78.66	78.71

Figure IA.1: Cross-sectional average of fund market betas

This figure depicts the cross-sectional average of fund market betas.



months within the estimation window. Then, for each fund mf, we compute the average alpha $\bar{\alpha}_{mf}$ using all estimated alphas from all the corresponding windows. We report cross-sectional statistics of funds' average alphas under both the traditional approach and the achievable approach, where we retain only those benchmark factors in the time-series regression for which the shortsale-constrained mean-variance portfolio assigns a positive weight.

Figure IA.1 confirms that fund exposure to systematic risk fluctuates significantly over time. Between 1985 and 2005, the average fund exhibited a market beta below one. However, in the lead-up to the Financial Crisis, the average market beta rose from approximately 0.95 in 2003 to 1.17 in 2007, before steadily declining to around 1.0 in the following years.

Table IA.2 presents cross-sectional statistics of funds' average alphas for the seven factor models listed in Table 2. The results are consistent with those in Table 5 from the main body of the manuscript. Across all factor models, the average achievable alpha is significantly lower than that obtained under the traditional approach. Accounting for time-varying fund exposure to benchmark factors through rolling windows further reduces the achievable alpha, even for the CAPM model. This finding is particularly relevant given that financial services firms, such as Morningstar, typically report alphas based on short windows of 36 or 72 months rather than using a fund's entire return history. In shorter windows, periods of market underperformance may lead to optimal strategies that involve shorting the market, which explains why the achievable alpha for CAPM in this setting is much smaller than the traditional alpha.

Table IA.2: Traditional and achievable mutual-fund alphas using rolling windows

This table reports cross-sectional statistics for the traditional and achievable average alphas with respect to the seven factor models listed in Table 2. We compute funds' average alphas using 36-month rolling windows to estimate a fund's alpha. To compute a fund's alpha, we require a minimum of 30 months within the estimation window. Then, for each fund mf, we compute the average alpha $\bar{\alpha}_{mf}$ using all estimated alphas from all the corresponding windows. We compute the traditional alpha by regressing the fund returns on all factors for each model and the achievable alpha on only those factors with a strictly positive weight in the shortsale-constrained mean-variance portfolio. We report the average alpha across funds, its t-statistic, percentiles of the cross-sectional distribution of fund alpha and the percentage of funds with positive alpha. Alphas are annualized and reported in percentage.

	CAPM	FFC	FF5	FF6	HXZ	HXZM	VANG
Panel A: Traditinal alpha with respect to long-short factors							
Average alpha	0.52	0.09	0.55	0.49	0.30	0.46	0.12
t-stat	9.46	2.03	9.91	9.11	5.88	9.39	2.40
10th percentile	-2.20	-2.10	-1.99	-1.93	-2.12	-1.82	-2.29
50th percentile	0.50	0.11	0.27	0.26	0.23	0.38	0.12
90th percentile	3.26	2.43	3.54	3.29	2.91	3.01	2.66
Percentage of funds with α >0	61.18	52.87	55.92	55.84	55.54	59.07	53.21
Panel B: Achievable alpha with respect to long-only factors							
Average alpha	0.21	-3.22	-3.38	-3.64	-3.24	-3.36	-3.34
t-stat	3.87	-40.51	-44.95	-46.54	-41.71	-42.48	-45.26
10th percentile	-2.36	-7.61	-7.39	-7.92	-7.43	-7.76	-7.20
50th percentile	0.24	-2.41	-2.73	-2.88	-2.48	-2.56	-2.89
90th percentile	2.71	0.34	0.02	-0.14	0.33	0.27	0.07
Percentage of funds with α >0	56.83	12.58	10.08	9.22	12.58	12.45	10.64

IA.3 Replicating factors with mutual funds

One challenge faced by shortsale-constrained investors is gaining exposure to traditional academic factors, such as HML, which rely on taking short positions. As highlighted by Johansson et al. (2025), the substantial leverage required by academic factors presents a significant obstacle when attempting to replicate their factor returns through ETFs and mutual funds. To address this issue, we explore alternative factor model constructions in which benchmark factor models consist of (i) long-only versions of academic factors and (ii) equal-weighted portfolios of funds that exhibit significantly negative exposures to these factors. Specifically, for each long-short academic factor, we form an equal-weighted portfolio of funds with statistically significant negative exposures—defined as having a t-statistic less than -2—on the given factor. For example, in the context of the FFC model, we approximate

Table IA.3: Time-series properties and Sharpe ratio of replicated factors

This table reports the time-series correlation between the replicated factor and the original factor, the annualized Sharpe ratio of the replicated factor, and the annualized Sharpe ratio of the original factor.

	SMB	HML	CMA	RMW	UMD	ME	IA	ROE
Correlation with original factor	0.35	-0.41	-0.36	-0.35	-0.22	0.19	-0.56	-0.26
Sharpe ratio replicated factor	0.64	0.58	0.62	0.60	0.64	0.32	0.53	0.64
Sharpe ratio original factor	0.60	0.45	0.47	0.38	0.27	0.55	0.42	0.34

a fund-based HML factor by aggregating the returns of funds whose estimated loading on the traditional long-short HML factor is both negative and statistically significant.

Table IA.3 reports the time-series correlation between the replicated factors and the original long-short factors, along with the annualized Sharpe ratio of the replicated and original factors. With the exception of the size factors SMB and ME, our replicated factors successfully achieve the intended negative exposures. However, the correlations between the replicated and original factors are generally low, which is consistent with the findings of Johansson et al. (2025), who highlight the difficulty of replicating academic factor returns using only tradeable funds.

We now evaluate mutual fund performance using augmented factor models that incorporate both the long-only versions of academic factors and the replicated factors. Table IA.4 reports cross-sectional statistics for both traditional and achievable fund gross alphas, based on the five factor models described in Table 2 that rely on long-short factors. Consistent with the analysis in the main body of the manuscript, average achievable alphas are substantially lower than average traditional alphas across all models. Furthermore, the proportion of funds generating a positive achievable alpha ranges from 11.29% for HXZM to 26.80% for FFC—considerably lower than the fraction of funds with a positive traditional alpha, which ranges from 46.40% for HXZ to 48.58% for FF5.

IA.4 Traditional and Achievable Alpha Correlation

Figure IA.2 depicts the time series of the cross-sectional correlation between traditional and achievable fund alpha across the seven factor models listed in Table 2. We find that these correlations fluctuate considerably over time for most models. The CAPM consistently exhibits the highest correlation, although this relationship weakened notably during major market disruptions such as the dot-com bubble of the early 2000s and the great financial

Table IA.4: Mutual-fund alphas with replicated factors

This table reports cross-sectional statistics for the traditional and achievable fund gross alphas using augmented factor models that incorporate both the long-only versions of academic factors and the replicated factors for the five factor models listed in Table 2 that require long-short factors. Panel A reports cross-sectional statistics for the traditional alpha, and Panel B for the achievable alpha, obtained by regressing fund returns on just those replicated factors with a strictly positive weight in the shortsale-constrained mean-variance portfolio (over the sample period for which we have return data for the fund). We report the average alpha across funds, its t-statistic, the time-weighted average alpha (where the weight is proportional to the length of the sample period for which we have return data for the fund), and its t-statistic. We also report percentiles of the cross-sectional distribution of fund alpha and the percentage of funds with positive alpha and t-statistic greater than two. Alphas are annualized and reported in percentage. Like Barras et al. (2022), we winsorize observations that are more than five times the inter-decile range (difference between the 90th and 10th percentiles) away from the median.

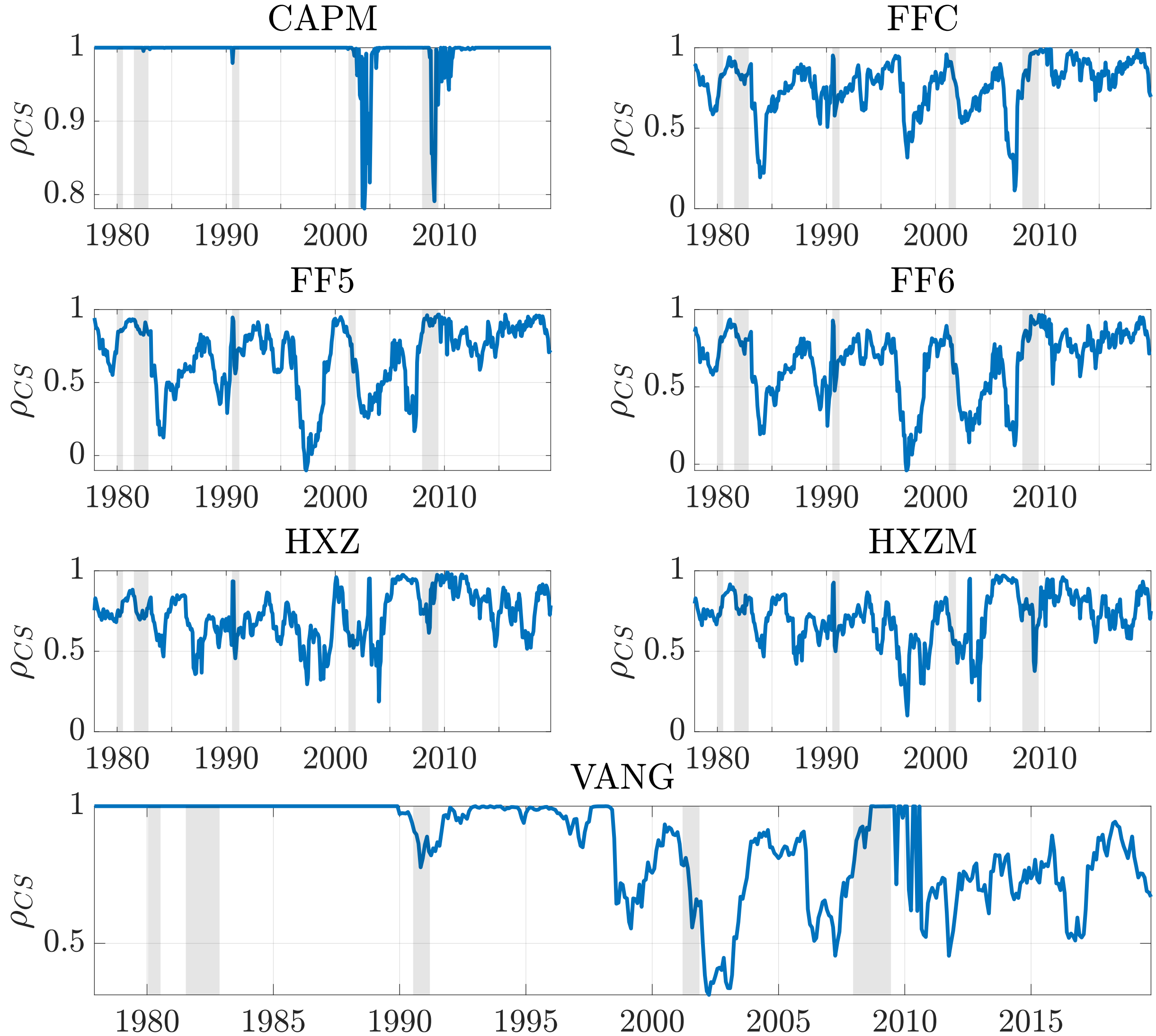
	FFC	FF5	FF6	HXZ	HXZM
Panel A: Fund-based factors without short-sale constraints					
Average alpha	-0.12	-0.15	-0.15	-0.17	-0.16
t-stat	-2.54	-3.23	-3.18	-3.25	-3.18
Time-weighted average alpha	0.06	0.06	0.03	-0.05	-0.02
t-stat	1.38	1.38	0.71	-1.06	-0.37
10th percentile	-2.64	-2.62	-2.65	-2.98	-2.87
50th percentile	-0.07	-0.05	-0.10	-0.15	-0.15
90th percentile	2.36	2.24	2.34	2.56	2.52
Percentage of funds with $\alpha > 0$	48.19	48.58	47.91	46.40	46.45
Percentage of funds with $t(\alpha) > 2$	9.27	8.19	9.00	9.05	9.22
Panel B: Fund-based factors with short-sale constraints					
Average alpha	-2.04	-2.13	-2.40	-3.30	-3.32
t-stat	-27.66	-31.91	-33.81	-44.82	-45.03
Time-weighted average alpha	-1.40	-1.49	-1.75	-2.76	-2.77
t-stat	-4.31	-4.33	-4.34	-4.36	-4.36
10th percentile	-6.05	-5.43	-6.12	-7.13	-7.15
50th percentile	-1.48	-1.62	-1.81	-2.69	-2.70
90th percentile	1.43	0.95	0.88	0.20	0.20
Percentage of funds with $\alpha > 0$	26.80	20.98	19.43	11.42	11.29
Percentage of funds with $t(\alpha) > 2$	1.98	1.38	1.42	0.34	0.47

crisis of 2009. The time-series average cross-sectional correlations for the CAPM, FFC, FF5, FF6, HXZ, HXZM, and VANG models are 99.41%, 76.29%, 67.56%, 65.57%, 72.47%, 70.90%, and 85.90%, respectively.

Figure IA.3 depicts the cross-sectional correlation between traditional CAPM alpha and the traditional fund alpha for the six non-CAPM models listed in Table 2, and Figure IA.4 depicts the cross-sectional correlation between traditional CAPM alpha and the

Figure IA.2: Cross-sectional correlation between traditional and achievable alpha

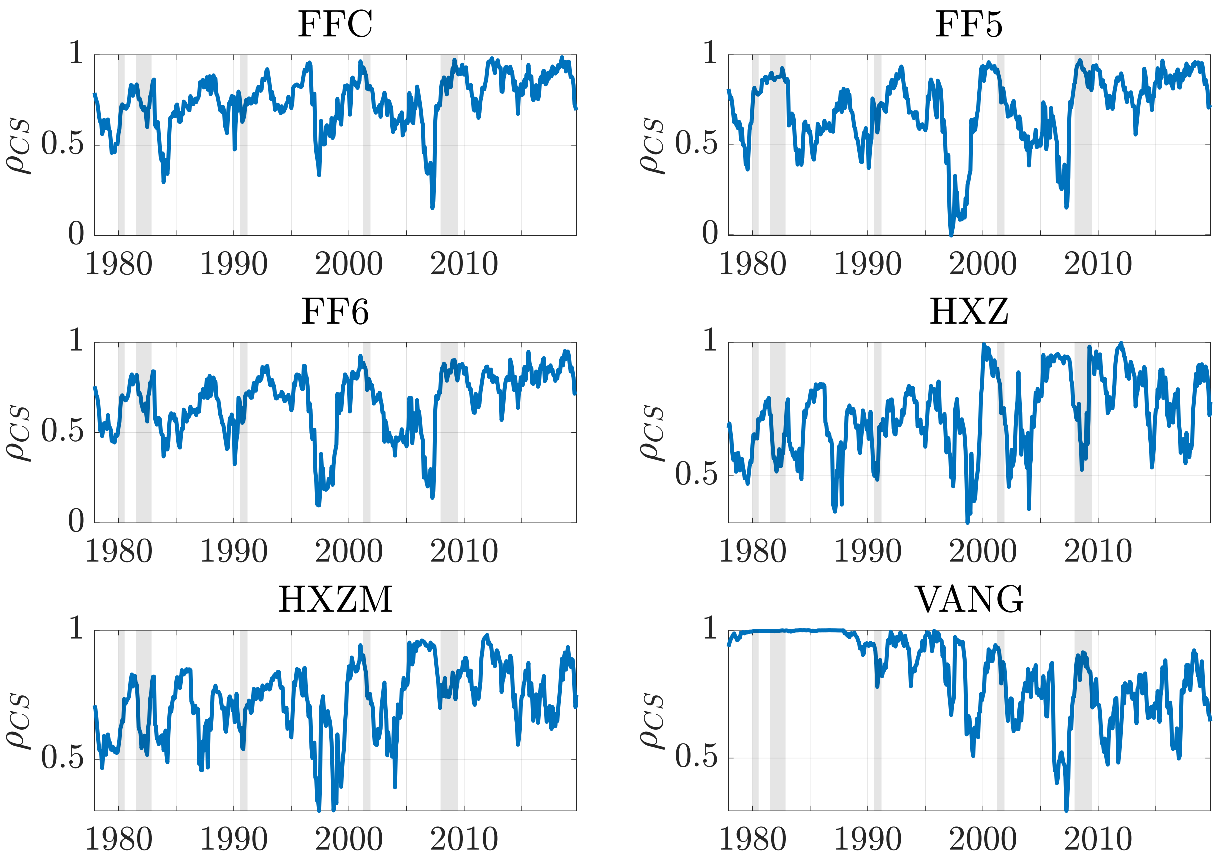
This figure depicts the cross-sectional correlation between traditional and achievable fund alpha for all the seven models listed in Table 2.



achievable fund alpha for the six non-CAPM models listed in Table 2. We observe that in general, the traditional CAPM alpha is much more correlated wiht the non-CAPM achievable alphas. In particular, the average correlation between traditional CAPM alpha and non-CAPM traditional alphas across all models is 73.32%, whereas the average correlation between traditional CAPM alpha and non-CAPM achievable alphas across all models is 94.60%.

Figure IA.3: Correlation between traditional CAPM and non-CAPM alphas

This figure depicts the cross-sectional correlation between traditional CAPM alpha and the traditional fund alpha for the six non-CAPM models listed in Table 2.



IA.5 Fund Flows: Controlling for CAPM alpha

In this Section, we repeat the analysis in Table 9 in the main body of the manuscript, controlling for traditional CAPM alpha for the pooled regressions in Panels C and D. We see in Table IA.5 that our results are robust to controlling for traditional CAPM alpha. One element to note from this analysis is that the significance of the coefficients for the achievable alphas in Panels C and D decreases substantially relative to that for the coefficients in Panels C and D in Table 9 where we do not control for traditional CAPM alpha. This is because, as we have seen in Figure IA.4, the correlation between traditional CAPM alpha and non-CAPM achievable alphas is large.

Figure IA.4: Traditional CAPM alpha and achievable non-CAPM alphas

This figure depicts the cross-sectional correlation between traditional CAPM alpha and the achievable fund alpha for the six non-CAPM models listed in Table 2.

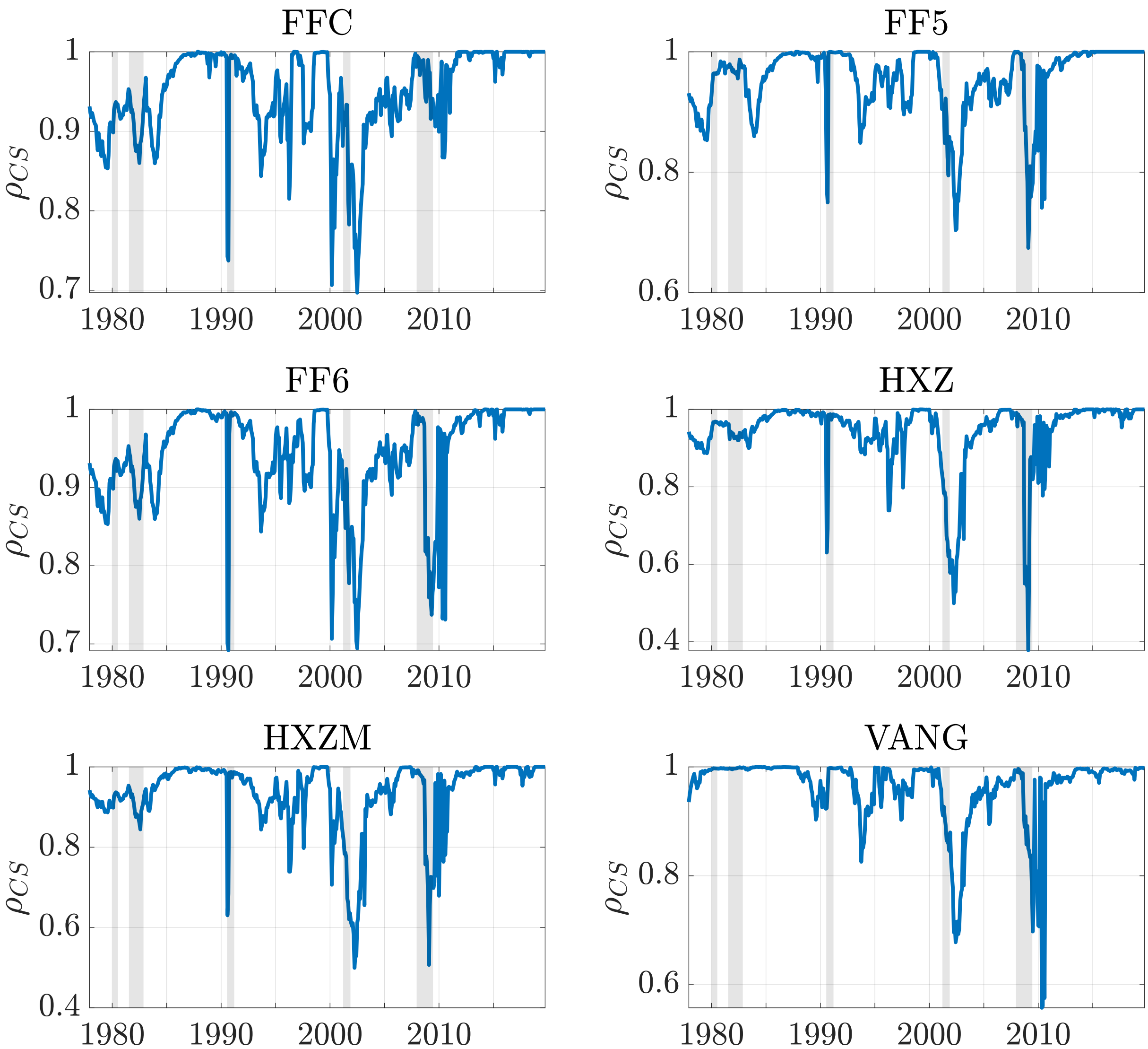


Table IA.5: Achievable and Traditional Alphas and Fund Flows

This table reports slope coefficients and their t-statistics for several versions of panel regression (11). Panel A considers the regression of fund flows on traditional alpha, Panel B the regression of fund flows on achievable alpha, Panel C the regression of fund flows jointly on traditional and achievable alphas, and Panel D the regression of fund flows jointly on traditional and achievable alphas and their interactions with a "Risk" indicator variable that takes a value of one for months in the top decile of months sorted by realized market volatility in the prior 36 months, and zero otherwise. For all regressions, we consider time and fund fixed effects, control for lagged values of fund flows up to 12 months, and double-cluster standard errors by time and fund. Finally, the regressions in Panels C and D include the traditional CAPM-alpha as a control in all models except for the CAPM.

	CAPM	FFC	FF5	FF6	HXZ	HXZM	VANG
Panel A: Traditional alpha							
Slope	0.162	0.083	0.054	0.049	0.066	0.071	0.031
	[29.561]	[18.946]	[11.891]	[11.018]	[8.363]	[14.457]	[5.128]
R2 (%)	2.639	2.956	2.790	2.778	2.851	2.893	2.678
Panel B: Achievable alpha							
Slope	0.159	0.148	0.153	0.149	0.155	0.152	0.146
	[21.728]	[21.203]	[21.559]	[21.435]	[21.585]	[21.628]	[19.980]
R2 (%)	2.544	2.182	2.350	2.233	2.408	2.320	2.117
Panel C: Traditional and achievable alpha							
Slope $\alpha_{\mathcal{T}}$	0.137	0.084	0.054	0.049	0.061	0.068	0.032
	[4.832]	[12.343]	[8.472]	[7.807]	[6.994]	[10.268]	[3.763]
Slope $\alpha_{\mathcal{A}}$	0.026	-0.007	0.035	0.021	0.039	0.032	-0.007
	[0.946]	[-0.514]	[2.345]	[1.588]	[2.958]	[2.492]	[-0.444]
R2 (%)	2.642	2.957	2.813	2.788	2.885	2.918	2.679
Panel D: Traditional and achievable alpha with risk-interaction terms							
Slope $\alpha_{\mathcal{T}}$	0.147	0.086	0.058	0.052	0.066	0.074	0.038
	[4.929]	[12.675]	[8.754]	[7.977]	[7.422]	[11.446]	[4.227]
Slope $\alpha_{\mathcal{A}}$	0.014	-0.008	0.033	0.021	0.035	0.029	-0.012
	[0.501]	[-0.577]	[2.233]	[1.539]	[2.649]	[2.265]	[-0.712]
Slope $\alpha_{\mathcal{T}}\times$Risk	-0.144	-0.014	-0.024	-0.017	-0.024	-0.031	-0.022
	[-3.695]	[-1.342]	[-3.136]	[-2.259]	[-2.724]	[-3.973]	[-2.057]
Slope $\alpha_{\mathcal{A}}\times$Risk	0.148	0.012	0.024	0.018	0.028	0.031	0.022
	[3.853]	[1.359]	[3.058]	[2.502]	[3.264]	[4.044]	[2.328]
R2 (%)	2.671	2.962	2.841	2.804	2.923	2.967	2.697